Design of concrete flexural and compressive members and calculation of maximum deflection and maximum crack width of members

Lixian Xu

College of Architectural Engineering, North China University of Technology, China.

Abstract

Under the action of vertical load, section 250mm×250mm, net span of 5 meters of reinforced concrete beam bending deformation.Reinforced concrete works together before concrete cracking, after concrete cracking, concrete exit work, the component enters the working stage with cracks until the steel bars yield, recording the maximum deflection and crack width in this state； Under vertical and lateral force, section 400mm×400mm, 3 meters high reinforced concrete column occurred large eccentric compression, calculating its maximum deflection and crack width.

Keywords

Reinforce concrete beam； reinforce concrete column； maximum deflection； max crack width.

1. Introduction

Typical test beams with normal section of flexural members are usually simple supported beams with rectangular section and single reinforcement. The beams should have sufficient shear resistance to ensure that no shear failure occurs in flexural experiments. The loading mode is concentrated load in mid-span, loading step by step, from zero to the bending failure of the normal section of the beam. At this time, the mid-span deformation is the largest, that is, the maximum deflection, but also the location of the maximum crack width. In practical engineering, the eccentricity of structural members may be increased due to the inaccuracy of load position, the non-uniformity of concrete and the deviation of construction. The horizontal and vertical loads on the top of reinforced concrete columns are adopted to simulate the eccentric compression and shear of columns that may occur in engineering practice. The maximum deflection and crack width of the column are calculated. Fig. 1 is the loading diagram of a beam under concentrated load, and Fig. 2 is the loading diagram of a column under horizontal and vertical loads.

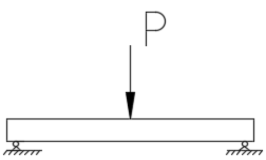


Figure 1 Beam Loading Diagram

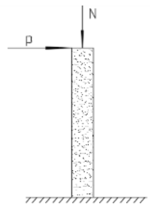


Figure 2 Column loading diagram

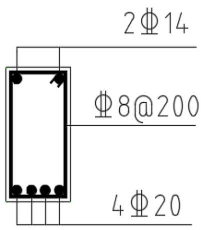


Fig. 3 Reinforcement of beam section

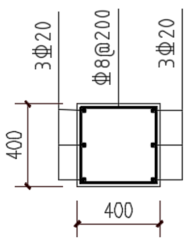


Fig. 4 Reinforcement of Column Section

2. Basic Component Parameters

C30 is used in concrete, and the cross-section size of the beam is 250mm×250mm, Net span 5 m, Thickness of protective layer C = 20mm, Longitudinal reinforcement and stirrups are HRB400 concrete material parameters： f_c=14.3N/mm^2, f_{ck}=20.1N/mm^2, f_t=1.43N/mm^2, f_{tk}=2.01N/mm^2, E_C=3.00x10^{4}N/mm^2, ho=460mm。Rebar material parameters: f_y=360N/mm^2, E_S=2.00x10^{5}N/mm^2。

3. Calculation of Maximum Deflection and Crack of Rectangular Beam with Single Reinforcement

3.1 Basic formula for calculating bearing capacity of normal section

$$\alpha_1 f_c b x = f_y A_s$$
$$M = f_y A_s (h_0 - \frac{x}{2}) \tag{1}$$

In formula: when the strength grade of concrete is less than or equal to C50, take alpha 1 = 1.0. The area of longitudinal tendons is known as As = 1256 mm2. From formula (1), the maximum bending moment and shear force can be calculated. The maximum bending moment is 179.507 kN m and the maximum shear force is 71.8 kN.

3.2 Basic Formula for Calculating Oblique Bearing Capacity

$$V_{cs} = \frac{1.75}{\lambda + 1} f_t b h_0 + f_{yv} \frac{A_{sv}}{s} h_0 \tag{2}$$

In the formula, lambda is the shear span ratio of the calculated section, lambda = a/h0, A is the distance between the point of action of concentrated load and the support section or the edge of the node, 1.5 when lambda < 1.5, and 3 when lambda > 3. The stirrups can be calculated by formula (2). See Figure 3 for section reinforcement

3.3 Stiffness formula

$$B_s = \frac{E_s A_s h_0^2}{1.15\Psi + 0.2 + \frac{6\alpha_E \rho}{1 + 3.5\gamma_f'}} \tag{3}$$

In the formula: Bs is short-term stiffness of reinforced concrete beams after cracking. Short-term stiffness is adopted because the long-term effect of load is not taken into account, i.e. the creep of concrete in compression zone is not taken into account. Ψ is the non-uniform strain coefficient of longitudinal tension steel bar between cracks. Take 0.2 when$\Psi < 0.2$ and 1.0 when $\Psi > 1.0$, For members directly subjected to repeated loads, Ψ=1.0; γ_f' is the strengthening factor of compressed flange; for rectangular section, γ_f'=0。Short-term stiffness of reinforced concrete beams after cracking can be calculated by formula (3) B_S=3.15×10^{13}N•mm^2

3.4 Formulas for Calculating Deformation and Crack Width

$$f = \frac{Pl^3}{48EI} \tag{4}$$

$$\omega_{max} = \alpha_{cr} \Psi \frac{\sigma_{sq}}{E_s} (1.9C_s + 0.08 \frac{d_{eq}}{\rho_{te}}) \tag{5}$$

In the formula, α_{cr} is the stress characteristic coefficient of members, for flexural and eccentric compression members, α_{cr} = 1.9; for eccentric tension members,α_{cr}=2.4; for axial tension members, α_{cr}= 2.7; C_S is the distance from the outer edge of the outermost longitudinal tension steel bar to the bottom of the tension zone, C_S= 20 mm when C_S<20 mm; and C_S= 65mm when C_S > 65 mm. Formula (4) calculates the maximum deflection of the span to be 11.872 mm, and formula (5) calculates the maximum crack width to be 0.414 mm.

4. Compression calculation of symmetrically reinforced columns

At the same time, the components bearing axial force and bending moment are eccentric compression members. The interaction of axial force N and bending moment M is equivalent to the eccentric pressure of an eccentric distance e_0=M/N. Therefore, eccentric compression members are not only one of the most basic and widely used components. As shown in Figure 2, reinforced concrete columns are subjected to horizontal force P and vertical axial pressure N, which are equivalent to eccentric compression. The bearing capacity of the column is checked under given axial compression ratio, longitudinal reinforcement and stirrups, and its maximum deformation and crack width are calculated. The reinforcement ratio of column section is shown in Figure 4. The unilateral reinforcement ratio of longitudinal reinforcement is 0.6%, i.e. As=942 mm2; the total reinforcement ratio of longitudinal reinforcement is 1.2%, within the required range (0.5%-5.0%).

4.1 Computation of N and e

Given the axial compression ratio of λ=0.4, the axial forces N and e are calculated. The basic formulas are as follows:

$$\lambda = \frac{N}{f_c A} \tag{6}$$

$$\begin{cases} N + f_y A_s = \alpha_1 f_c bx + f_y' A_s' \\ Ne = \alpha_1 f_c bx(h_0 - \frac{x}{2}) + f_y' A_s'(h_0 - a_s') \end{cases} \tag{7}$$

4.2 Discrimination of Size Eccentricity in Symmetrical Reinforcement

$$\begin{cases} \mathrm{e} > 0.3h_0 \\ \xi \le \xi_b \end{cases} \tag{8}$$

By substituting the result of formula (7) into the column, it can be judged that the column is subjected to large eccentric compression.

4.3 Calculation of Horizontal Force P

The bending moment M of the column can be obtained from formula (7), and the equivalent horizontal force P can be obtained from formula M. The basic formulas are as follows:

$$\begin{cases} e = e_i + \frac{h}{2} - a_s \\ e_i = e_0 + e_a, e_a = \max\{20, \frac{400}{30}\} \\ e_0 = \frac{M}{N} \\ M = Pl \end{cases} \tag{9}$$

From the formula, M = 200.429 kN m, P = 67 kN.

4.4 Checking calculation of stirrups

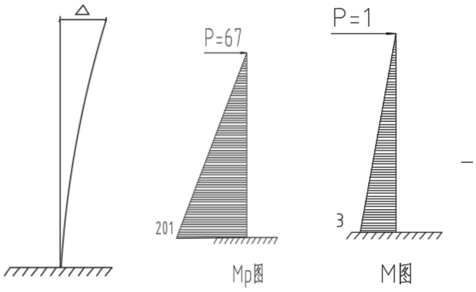
In the case of given stirrup reinforcement, it is necessary to verify the maximum shear force that the column can withstand. Under the action of pressure, the shear capacity of the two ends of the column is improved to a certain extent, and the quantified value is 0.07N (Nmax is 0.3fcA).

$$\begin{cases} V = \frac{1.75}{\lambda + 1} f_t b h_0 + f_{yv} \frac{A_{sv}}{s} h_0 + 0.07N \\ \lambda = \frac{H_n/2}{h_0} \\ N = 0.3 f_c A \end{cases} \tag{10}$$

The maximum shear force that the column can bear is 204kN, which is greater than the horizontal force P, so the shear capacity of the column meets the requirements.

4.5 Calculation of Maximum Deformation and Crack Width

As shown in Fig. 5 (a), the maximum deformation of the column occurs at the top of the column. The magnitude of the deformation can be obtained by multiplying the structural mechanics diagram, as shown in Fig. 5 (b).



3.5.1 Short-term stiffness B s calculation

Short-term stiffness is adopted for members without considering the long-term effect of load, i.e. the creep of concrete in compression zone.

$$B_s = \frac{E_s A_s h_0^2}{1.15\Psi + 0.2 + \frac{6\alpha_E \rho}{1+3.5\gamma_f'}} \tag{11}$$

Short-term stiffness of reinforced concrete beams after cracking can be calculated by formula (3) $B_S=1.76\times10^{13}N\bullet mm^2$.

3.5.2 Deflection formula

$$\Delta = \frac{Ay_0}{EI} \tag{12}$$

Formula A is the area of any moment graph, Y0 is taken from the center of another figure corresponding to the height of the straight line graph. The flexural stiffness of EI member, i.e. the short-term stiffness of this member, is Bs. The maximum deflection is 34.26 mm, and the maximum deflection appears at the top of the column.

3.5.3 Maximum crack width formula

$$\omega_{max} = \alpha_{cr}\psi\frac{\sigma_{sq}}{E_s}(1.9C_s + 0.08\frac{d_{eq}}{\rho_{te}}) \tag{13}$$

The maximum crack width calculated by formula (14) is 0.54 mm.

5. Epilogue

By combining the principles of normal section design and oblique section design of members in the basic principles of concrete with the basic methods of structural mechanics, the rationality of reinforcement of members can be checked, the location and size of maximum deflection and maximum cracks can be calculated, and the conclusions drawn from specific experiments can provide some reference for engineering practice.

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