Exact solutions of (3+1)-dimensional Kadomtsev-Petviashvili Equation

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Abstract

With the aid of symbolic computation software Maple, we use the Riccati equation method to study a reduced equation of (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation. Consequently, we obtain the exact solutions in various forms of the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation. This method is very effective for solving high dimensional differential equations, and can obtain rich exact solutions.

Keywords

Exact solution, (3+1)-dimensional Kadomtsev-Petviashvili equation, Riccati equation method.

1. Introduction

In practice, exact solutions of nonlinear differential equations can help physicists and engineers to study the propagation law of waves, it provides accurate data for testing the accuracy of numerical solutions, so finding explicit exact solutions of nonlinear differential equations is always one of the most important problems in the field of differential equation research. After a long-time effort, a lot of work has been done on solving nonlinear differential equations, and proposed many ingenious effective methods, such as inverse scattering method, Bäcklund and Darboux transformation method, Hirota bilinear method, function expansion method [1,2], Tanh function method [3] and Riccati equation method [4] and so on.

In this paper, the exact solutions of (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation is studied by using the Riccati equation method.

The form of (3+1)-dimensional KP equation is as follows[5]

$$(u_t+6uu_x+u_{xxx})_x-3u_{yy}-3u_{zz}=0, \tag{1}$$

where $u=u(x,y,z,t)$, the subscripts denote partial derivative. (3+1)-dimensional Kadomtsev-Petviashvili (KP) equations are generally used to describe solitons and nonlinear wave dynamic systems in the field of fluid dynamics [6]. Use the homogeneous balance method, Khalfallah[7] obtain many solition-like solutions and periodic-like solutions of this equation.

Since it is difficult to solve (3+1)-dimensional partial differential equations directly, we can first reduce Eq. (1), find the solutions of the reduced equation, and combine the similar variables, finally we can get the group-invariant solutions of the original equation with any function. According to the literature [8], we have the following changes to Eq. (1):

$$u(x,y,z,t)=F(x,y,\tilde{z}),\tilde{z}=t-\frac{z}{6}. \tag{2}$$

Substitute Eq. (2) into Eq. (1), so Eq. (1) is reduced to:

$$F_{x\tilde{z}}+6F_x^2+6F_{xx}F+F_{xxxx}-3F_{yy}-\frac{1}{12}F_{\tilde{z}\tilde{z}}=0. \tag{3}$$

2. Exact solutions

Let the traveling wave solution form of Eq. (1) be:

$$F(x,y,z)=v(\eta), \qquad \eta=kx+ly+m\tilde{z}, \tag{4}$$

where k,l,m are constants. Substitute Eq. (4) into Eq. (3), then Eq. (3) can be reduced to the following nonlinear ordinary differential equation

$$kmv''+6k^2v'^2+6k^2vv''+k^4v''''-3l^2v''-\frac{m^2}{12}v''=0. \tag{5}$$

Now find the solution of Eq. (5) that has the following form

$$v(\eta)=\sum_{i=0}^{m}a_i\phi^i, \tag{6}$$

where a_i are constants to be determined later, ϕ satisfies the Riccati function

$$\phi'=\phi^2+r, \tag{7}$$

where r is a constant. Balancing the highest order linear term v'''' and the highest order nonlinear term v'^2, it is easy to know $m=2$. Therefore,

$$v(\eta)=a_0+a_1\phi+a_2\phi^2. \tag{8}$$

Substituting Eq. (8) and Eq. (7) into Eq.(5), and equating the coefficients of $\phi^i (i=0......6)$ to zero, we can get a system of a_0,a_1,a_2,k,l and m. The solutions of the system can be obtained by means of Maple,

$$a_0=-\frac{1}{72}\frac{96k^4r+12km-36l^2-m^2}{k^2}, a_1=0, a_2=-2k^2. \tag{9}$$

The Riccati equation (7) has the following solutions[8]:

$$\phi=\begin{cases}-\sqrt{-r}b\tanh(\sqrt{-r}\eta), & r<0\\ -\frac{1}{\eta}, & r=0\\ \sqrt{r}b\tan(\sqrt{r}\eta), & r>0\end{cases} \tag{10}$$

where $\eta=kx+ly+m\tilde{z}$.

Then the exact solutions of Eq. (3) are as follows:

$$F(x,\tilde{y},z)=\begin{cases}-\frac{1}{72}\frac{96k^4r+12km-36l^2-m^2}{k^2}+2b^2k^2r\tanh^2(\sqrt{-r}\eta), & r<0\\ -\frac{1}{72}\frac{12km-36l^2-m^2}{k^2}-\frac{2k^2}{\eta^2}, & r=0\\ -\frac{1}{72}\frac{96k^4r+12km-36l^2-m^2}{k^2}-2b^2k^2r\tan^2(\sqrt{r}\eta), & r>0\end{cases} \tag{11}$$

where $\eta=kx+ly+m\tilde{z}$.

So the solutions to the (3+1)-dimensional KP equation (1) are

$$\mathrm{u}(x,y,z,t)=\begin{cases}-\frac{1}{72}\frac{96k^4r+12km-36l^2-m^2}{k^2}+2b^2k^2r\tanh^2(\sqrt{-r}\eta), & r<0 \quad (a)\\ -\frac{1}{72}\frac{12km-36l^2-m^2}{k^2}-\frac{2k^2}{\eta^2}, & r=0 \quad (b)\\ -\frac{1}{72}\frac{96k^4r+12km-36l^2-m^2}{k^2}-2b^2k^2r\tan^2(\sqrt{r}\eta), & r>0 \quad (c)\end{cases} \tag{12}$$

where $\eta = kx + ly + m(t - \frac{z}{6})$.

Now let us consider the solutions (12) of the KP equation (1) along $y = 0, z = 0$.

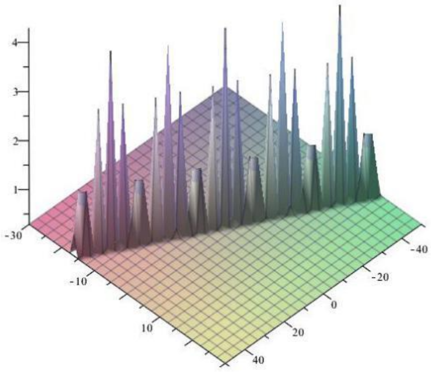

Fig 1: A solution (12-a) with $k = 1, l = 2, m = 3, r = -2$.

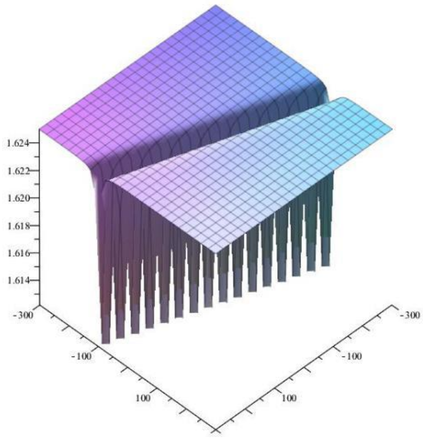

Fig 2: A solution (12-b) with $k = 1, l = 2, m = 3, r = 0$.

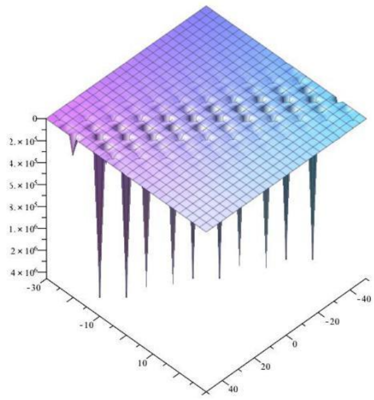

Fig 3: A solution (12-c) with $k = 1, l = 2, m = 3, r = 2$.

3. Conclusion

It is difficult to obtain rich exact solutions of (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation directly, the calculation process is often complicated. This paper studies a reduced equation of (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation, and get the exact solutions and numerical examples of the original equation by means of the Raccati equation method.

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