A new hybrid reliability analysis method based on simulated annealing external penalty function method

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Abstract

To analyze the influence of parameter uncertainty on structural reliability and solve the problem of penalty factor selection in penalty function, a hybrid reliability analysis method based on simulated annealing external penalty function method is proposed in this paper. Firstly, the reliability index is regarded as the objective function and the limit state equation is regarded as the equality constraint by using the optimization theory. Then the mathematical model of the reliability optimization design of the structure is established. Secondly, the simulated annealing external penalty function method is used to transform the constrained optimization problem into an unconstrained optimization problem. Considering the uncertainty of parameters, a probabilistic-interval mixed uncertainty model is established by using interval variables. Based on this, the relationship between parameter uncertainty and reliability index and the boundary value of reliable index of interval variable are studied in this paper. Numerical examples and engineering examples show that the simulated annealing external penalty function method has good convergence and high computational accuracy for certain nonlinear functions. The uncertainty of reliability parameters has a great influence on the reliability of structures. There is a positive correlation between the uncertainty of parameters and the uncertainty of reliability index.

Keywords

Uncertainty; Simulated annealing external penalty function; Hybrid reliability; Interval variable.

1. Introduction

In the practical engineering structure design, there are often various uncertainties such as material properties, manufacturing error, installation errors, and its own environment. Also, Recently, numerous research accomplishments of many advanced computational methods have been developed to sole the pratical engineering problems [1-4]，especially in the field of structural reliability. In order to improve the product performance and structural reliability [5], lots of advanced design and analysis methods based on uncertainty have attracted the attention of many researchers.There are great limitations in the reliability analysis methods which are applied to traditional probability models. They can only be performed with sufficient parameter samples and a strict probability distribution model. Therefore, it is difficult to calculate and analyze many uncertainties in practical engineering problems. When the parameter sample value itself has a certain fluctuation range and uncertainty, it is of great practical significance to study the structural hybrid reliability. Xiao et al. [6] proposed a structural reliability analysis method under mixed uncertainty. Aiming at the complex non-linear relationship between variables and failure modes, the maximum and minimum failure probability of each failure mode can be calculated, and the structural reliability analysis is carried out by obtaining the corresponding optimization model; Jiang et al. [7] proposed an efficient probabilistic-evidence mixed reliability analysis method which aims at the mixed uncertainty problem of fuzzy variables and evidence variable structures, and this method greatly improves the calculation efficiency under the premise of ensuring accuracy. Meng et al. [8] proposed a new structural hybrid reliability analysis method based on dimensionality reduction algorithm, and the reliability of the structure is evaluated by obtaining the membership degree of the failure probability. Kang [9] proposed a reliability-based structural optimization design method by modeling the mixture of the probability and convex set of uncertain parameters. Li [10] proposed a dynamic structure reliability analysis method under the condition of mixed uncertain parameters. Considering the uncertainty of the parameters in the structural dynamic reliability analysis model, the corresponding dynamic reliability was calculated, and the influence of the parameter type on the dynamic reliability was analyzed; Wei et al. [11] proposed a core structure reliability analysis method of the hybrid machine based on the discrete element method. During the mixing process, the introduction of the discrete element method is feasible and operable, and can also ensure the safety and reliability of the institution. Jiang et al. [12] proposed a probability interval mixed uncertainty model and structural reliability analysis method considering correlation, and this method solved the mixed reliability analysis problem of correlation between variables. It is of great practical significance to be able to deal with the mixed reliability analysis problem with the correlation between variables. This method gives the interval of reliability index and the failure probability, and solves the case of variable correlation constraints in practical engineering problems. Meng et al. [13] proposed a structural hybrid reliability analysis method based on Taylor expansion method. For the mixed uncertainty problem of random variables and interval variables, the upper and lower bounds of the structural failure probability were calculated by Taylor expansion. The research on hybrid reliability methods has attracted the attention of many scholars at home and abroad. Adduri [14] proposed a method for analyzing the reliability limits of interval variable structure systems based on approximate joint function functions. Qiu et al. [15 combined classical probability theory and interval algorithms to study the structural reliability problems of probability interval structural systems. Wang et al. [16] proposed a reliability solution method for probability interval hybrid systems based on interval reliability models and probability calculations. Most of these methods mentioned above are aimed at simple linear problems. Considering the effects of various uncertainties in engineering practice, it is of great engineering significance to study and establish advanced methods to deal with mixed uncertainties.

The external penalty function method is an earlier optimization method that can effectively solve constrained optimization problems. The basic idea is to transform it into a penalty function which contains an objective function according to the characteristics of the constraint conditions, so that the constraint optimization problem is transformed into a series of unconstrained optimization problems. However, this method has its own shortcomings. Only when the penalty factor approaches infinity, the optimal solution obtained by the external penalty function method can approach the optimal solution of the original problem. However, as the penalty factor approaches infinity, there is a certain possibility that the behavior of the penalty function will be deteriorated and it will result in low accuracy of the calculation result and inaccurate reliability. Therefore, other algorithms (such as FOSM method, SQP methodand MCS method) are more likely to be used to solve the constraint optimization problem. It can be found through research that the main reason for the low accuracy and reliability of the penalty function is that when the penalty factor approaches infinity, the condition number of the Hessen matrix of the penalty function will be deteriorated. Then the unconstrained algorithm based on gradients cannot get the optimal solution. The simulated annealing algorithm (SA) does not need any gradient information in the entire calculation process. Therefore, the deterioration of the condition number of the penalty function Hessen matrix does not affect its calculation. In addition, for the same reason, during the process of the entire calculation, there is no need to gradually increase the penalty factor in order to approach the optimal solution. Instead, only a larger penalty factor needs to be taken directly, so that the selection of the penalty factor can be effectively solved.

Based on this, this paper proposes a new hybrid reliability analysis method based on simulated annealing external penalty function. By introducing the uncertainty of parameters and combining the probability model [17] to study the problem of mixed uncertainty in practical applications, the reliability index of the function of the structure under a certain degree of uncertainty [18] is mainly studied. The convergence problem of the solution, the relationship between the parameter uncertainty and the reliable index in the practical engineering problem, and the problem of parameter selection corresponding to the upper and lower bounds of reliability index are studied. The test function and engineering examples are used to verify the stability and effectivity of the proposed method.

2. Basic theory

This section introduces the structural reliability mathematical model and optimization algorithm theory. We will improve it based on the external penalty function and apply it to the structural reliability analysis.

2.1 Mathematical model of structural reliability

$X_1, X_2, \cdots, X_n$ are n independent random variables with arbitrary distributions. These independent random variables can constitute the structural limit state equation expressed as follows:

$$Z = g(X_1, X_2, \cdots, X_n) = 0 \tag{1}$$

Using $R-F$ (Lakovitz Fesley's method) to normalize the non-normal variable equivalents, we can get the equivalent normal mean σ'_{xi}, standard deviation μ'_{xi} and reliability indicators β:

$$\sigma'_{xi} = \phi\left\{\Phi^{-1}\left[F_{xi}\left(x_i^*\right)\right]\right\} / f_{xi}(X_i^*) \tag{2}$$

$$\mu'_{xi} = x_i^* - \Phi^{-1}[F_{xi}(x_i^*)]\sigma'_{xi} \tag{3}$$

$$\beta = (\sum\left[(X^*_i - \mu'_{xi}) / \sigma'_{xi}\right]^2)^{1/2} \tag{4}$$

As the check point is unknown, it can use β as a function $P(X_1, X_2, \cdots, X_n)$ of the points on the surface of the limit state, and the corresponding optimization method is used to obtain the minimum value β, and then the reliability index β and the check point $P^*(X_1^*, X_2^*, \cdots, X_n^*)$ can be obtained. The reliability index solution is transformed into the following constraint optimization model:

$$\beta = (\sum_i^n\left[(X^*_i - \mu'_{xi}) / \sigma'_{xi}\right]^2)^{1/2} \tag{5}$$

$$s.t. \quad Z = g(X_1^*, X_2^*, \cdots X_n^*) = 0 \tag{6}$$

If one variable (X_j) in the limit state equation can be expressed by other variables as:

$$X_j = g'(X_1, X_2, \cdots, X_{j-1}, X_{j+1}, \cdots, X_n), \tag{7}$$

then the constraint optimization model (6) can be expressed as (8):

$$\min \quad \beta^2 = \sum_{i=1, i\neq j}^{n}\left[(X_i^* - \mu'_{xi}) / \sigma'_{xi}\right]^2 + \left\{\left[g'(X_1^*, X_2^*, \cdots X^*_{j-1}, X^*_{j+1}, \cdots X^*_n) - \mu'_{xj}\right] / \sigma'_{xi}\right\} \tag{8}$$

2.2 Optimization algorithm

2.2.1 The Simulated Annealing Algorithm

The idea of Simulated Annealing (SA) was first proposed by Metropolis et al. in 1953. In 1983, Kirkpatrick first applied the simulated annealing algorithm to solve the optimization problems [18]. The simulated annealing algorithm is a stochastic optimization algorithm based on Monte Carlo iterative solution strategy. It utilizes the common ground between the physical solid annealing process and the optimization problem, and its ultimate purpose is to provide an effective approximate solution algorithm for non-deterministic polynomial (NP) complexity problems. The algorithm can overcome the defect about easily falling into local minimum and the laziness of initial value.

Simulated annealing is an optimization algorithm extended by the local search algorithm. It is different from the local search algorithm in that it selects a poor solution with a large target value in the field with a certain probability. In theory, it is a global optimal algorithm. The simulated annealing algorithm is based on the similarity between the solution process of the optimization problem and the annealing process of the physical system. The Metropolis algorithm is fully used to properly control the cooling process to achieve simulated annealing and then solve the global optimization problem.

The basic principle of the simulated annealing algorithm is as follows:

Firstly, given the initial temperature T_0 and initial point x_0, and then calculate the function value at that point $f(x_0)$.

Secondly, we randomly generate variables Δx, get the updated status $x' = x + \Delta x$, and calculate the function value under this status $f(x')$ and the difference $\Delta f = f(x') - f(x)$.

If $\Delta f \le 0$, use this state as the initial point of the next simulated annealing.

If $\Delta f > 0$, calculate the acceptance probability of the new state $P(\Delta f) = \exp(-\Delta f / T)$, and generate a uniformly distributed pseudo-random number c in the interval $[0,1]$. If $P \le c$, this state is used as the initial point for the next simulated annealing; otherwise, the original state is used as the starting point without change.

The above steps are the Metropolis process, and the temperature is reduced according to a certain annealing scheme, and the above process is repeated until the end criterion is reached. The algorithm can converge to the global best advantage or approximate the global best advantage.

2.2.2 The optimized mathematical model of simulated annealing external penalty function method

In this section, we will apply the simulated annealing algorithm to solve the structural reliability analysis. The sudden jump of the algorithm can effectively avoid the dilemma of local optimal solution. The combination of simulated annealing algorithm and external penalty function method can be used to solve the structural reliability analysis problem with certain nonlinearity.

The external penalty function method can approximately convert a constrained optimization problem into an unconstrained optimization problem according to the type of constraints. In a general constrained optimization problem, the objective function is given as:

$$\min\ f(X_1, X_2 \cdots X_n) \tag{9}$$

the restrictions are as follows:

$$\text{s.t. } h_i(X_1, X_2 \cdots X_n) = 0 \quad i \in E = \{1, \dots, l\} \tag{10}$$

$$g_i(X_1, X_2 \cdots X_n) \le 0 \quad i \in I = \{1, \dots, m\} \tag{11}$$

The feasibility range is given by: $D = \{X \in R^n | h_i(X) = 0 (i \in E), g_i(X) \le 0 (i \in I)\}$. Then we construct a penalty function:

$$\overline{P}(X) = \sum_{i=1}^{l} h_i^2(X) + \sum_{i=1}^{m} \max\{0, g_i(X)\} \tag{12}$$

The following augmented objective function is obtained as:

$$F(X, M_k) = f(X) + M_k [\sum_{i=1}^{l} h_i^2(X) + \sum_{i=1}^{m} \max\{0, g_i(X)\}^2] \tag{13}$$

Among them, $\{M_k\}$ is a monotonically decreasing positive sequence, i.e., $M_1 > M_2 > \cdots > M_k$. $f(X)$ is an objective function without penalty term; $\overline{P}(X)$ is a penalty function; M_k is a penalty factor, and $M_k\overline{P}(X)$ is a penalty term. For the points $\overline{P}(X)$ that do not satisfy the constraint condition X, the penalty term $M_k\overline{P}(X) > 0$ decreases with M_k decreasing, which is a kind of punishment when the X constraint conditions are not satisfied. When the $M_k\overline{P}(X) = 0$ constraints are met, the penalty term indicates impunity.

The steps of the hybrid algorithm combining the simulated annealing algorithm SA and the external penalty function method are as follows:

For the structural optimization problem, the objective function is selected as the objective function with a penalty term. The method $\{M_k\}$ is used to generate the decreasing $M_{k+1} = r * M_k$ sequence. The decreasing coefficient is given by $r \in [0,1]$. The accuracy is given by $\varepsilon > 0$, and the annealing factor is given by q. Take a larger number of initial penalty factors as M_0 and leave them unchanged. (Here we choose $M_0 = 1e8$， $r = 0.96$， $\varepsilon = 1e\text{-}6$， $q = 1$)

Arbitrarily select an initial solution (initial state) $X^{(0)}$ order $k = 0$, $X^{(k)} = X^{(0)}$ and $T_k = T_{\max}$ (initial annealing temperature) calculated $F(X^{(k)}, M_k)$ values.

Do the following cycle at temperature T_k.

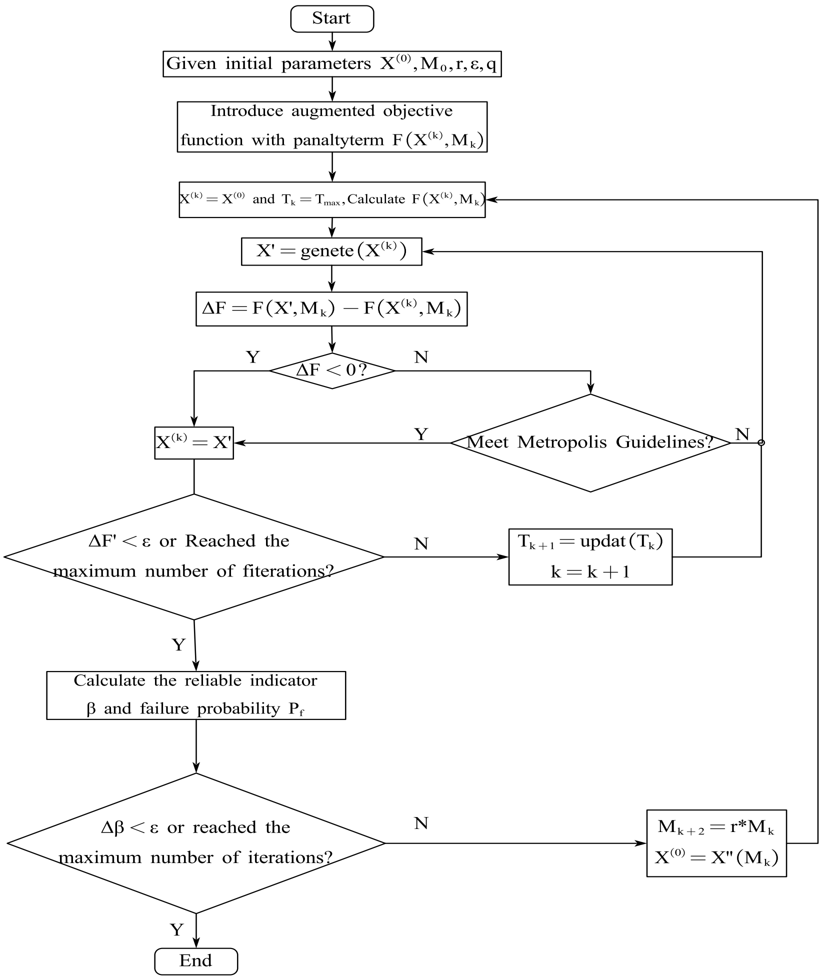

Fig. 1 Flow chart of the algorithm

Generate a new state randomly $X' = genete(X^{(k)})$ (candidate solution) at the current temperature T_k;

Calculate the augmented objective function value $F(X^{(k)}, M_k)$ and $\Delta F = F(X', M_k) - F(X^{(k)}, M_k)$;

If $\Delta F \le 0$, so $X^{(k)} = X'$, go to 4), otherwise go to the next step;

If the condition $\min\{1, \exp(-\Delta F / T_k)\} > random(0,1)$ is satisfied, then go to 4), otherwise go to the step a.（Metropolis);

If the convergence accuracy of the algorithm is satisfied or the upper limit of the number of iteration steps is reached, go to 5); Otherwise, the annealing temperature of the next cycle is obtained by $T_{k+1} = updat(T_k)$ and $k = k+1$, go to 3). (Herein we take $T_{k+1} = \lambda T_k, \lambda \in [0,1]$ and use the exponential cooling method to update the temperature)

Output the final calculation result, $X^* = X'$ and $F(X^{(k)}, M_k) = F(X', M_k)$ take the sum.

In this paper, a hybrid algorithm combining the simulated annealing algorithm and the external penalty function method is used to give the corresponding functional equation and the distribution profile of its variables. According to the above optimization theory, the reliability can be obtained by combining the formula β and the formula x^*. The reliability index can be iteratively obtained by the simulated annealing external penalty function method. The flowchart of the proposed method is shown in Fig. 1.

3. Reliability analysis

3.1 Numerical example

The function expression of structure of the structure is given as：

$$g = 5000 - x_1^2 \cdot y_1 - x_2^2 \cdot y_2^2 \tag{14}$$

In Eq. (14), the distribution types of variables x_1, x_2, y_1, y_2 are normally distributed and the variables are independent each other. The three parameters of random variables are listed below, calculated and analyzed.

（1)We consider the uncertainty of the mean value of variables X and Y 10%. At this time, the parameter values are shown in Table 1.

Table 1 The value of each random variable distribution parameter

Random Variables	Mean(μ)	variance(σ)	Distribution type
x_1	[3.6,4.4]	0.6	Normal
x_2	[4.5,5.5]	0.5	Normal
y_1	[36,44]	2.5	Normal
y_2	[9,11]	0.8	Normal

For the above parameters, the upper and lower bounds of the reliability index are calculated and analyzed.The cumulative probability function of normal distribution :

$$cdf = \frac{1}{2} + \frac{1}{2} erf\left[\frac{x-\mu}{\sqrt{2}\sigma}\right], \tag{15}$$

in which, define domain： $x \in R$ and the parameter range is given by： $\mu \in R$， $\sigma^2 \ge 0$。

1)By the partial derivation of parameters μ ：

$$\frac{\partial cdf}{\partial \mu} = -\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2\right] \tag{16}$$

According to the variable interval of this example, we can get $\frac{\partial cdf}{\partial \mu} < 0$. Therefore, the cumulative probability function decreases monotonically to the parameter μ .

2)Through the partial derivation of the parameters σ , we can get:

$$\frac{\partial cdf}{\partial \sigma} = -\frac{x-\mu}{\sqrt{2\pi\sigma^2}} \exp\left[-\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2\right] \tag{17}$$

It can be seen from the above formula (17) that the positive and negative of $\frac{\partial cdf}{\partial \sigma}$ depends on the size relationship with $\frac{\partial cdf}{\partial \sigma}$:

If $x<\mu$, $x-\mu<0$, then $\frac{\partial cdf}{\partial \sigma}>0$, and the cumulative probability distribution function increases with σ monotonically;

If $x>\mu$, $x-\mu>0$, then $\frac{\partial cdf}{\partial \sigma}<0$, and the cumulative probability distribution function decreases with σ monotonically;

When we choose $x=\mu$, no matter what value σ is taken, the cumulative probability distribution function value is 0.5. According to the monotonicity analysis above, the lower limit of reliability index $\beta_{\min}$ will be reached at the upper limit of mean interval, and the upper limit of reliability index $\beta_{\max}$ will be reached at the lower limit of mean interval. The results are in good agreement with the analysis of the monotonicity. Using the proposed method in this paper, the improved first order second moment method and Monte Carlo iterative calculation, the reliability index calculation results and failure probability solution data are obtained as Table 2. The corresponding reliability index iteration diagram is obtained by the proposed method in 50 steps, as shown in Fig. 2 and Fig. 3.

Table 2 The calculation result of reliability index and failure probability

Method	Method of this article	FOSM	Monte Carlo
Reliable indicator interval	[0.5344, 4.0334]	[0.5291, 4.0264]	[0.5335, 4.0326]
Failure probability interval	[2.748×10^{-5}, 0.2966]	[2.8323×10^{-5},0.2984]	[2.7585×10^{-5}, 0.2968]

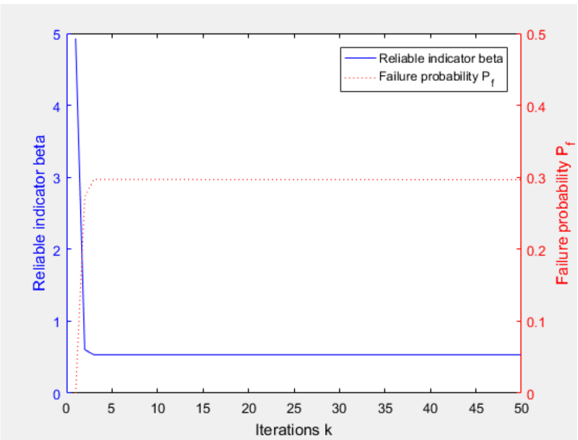

Fig. 2 The iteration of lower bound of reliability index

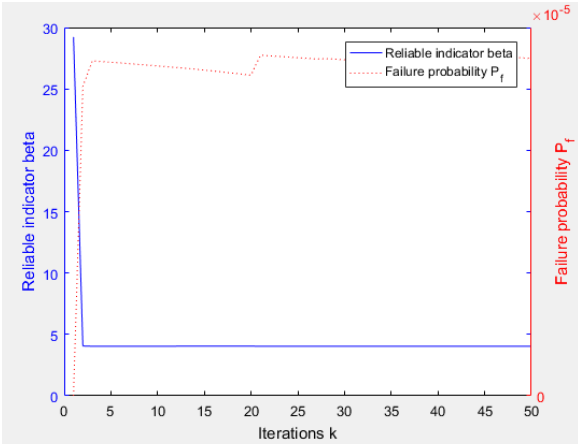

Fig. 3 The iteration of upper bound of reliability index

The calculation results in Table 2, Fig. 2 and Fig. 3 show:

(1) In the case of considering the uncertainty of the mean of the variables, the proposed method in this paper can obtain the reliable index $\beta_{\min}$ and $\beta_{\max}$ in 50 steps. The corresponding structural failure probability are $P_f = 0.2966$ and $P_f = 2.748\times10^{-5}$, respectively. The calculation results above are consistent with the general rule that the larger the reliability index β, the smaller the failure probability P_f.

(2) The calculation results using the improved first-order second-moment method (FOSM) also basically accord with the above rules. The Monte Carlo method is also adopted for the verification. When the number of samplings is N = 10^6, the calculation of the structural reliability index is also in line with the above law.

(3) It can be known from the calculation results of the above three methods that the fluctuation of the distribution parameters has a greater influence on the reliability probability or failure probability of the structural members; the failure probability fluctuates in a large range, but is relatively small, and the reliability of the structure is high.

(4) The method in this paper finds that the reliable index converges quickly in the first 10 iterations, but converges slowly in the next 40 iterations.

Considering the uncertainty of the mean X and Y of the sum of variables, the reliable index is calculated by the method in this paper. The reliable index interval and its uncertainty are shown in Table 3. The relationship between the mean uncertainty and the uncertainty of the reliable index is shown in Fig. 4.

Table 3 The computational results of reliable indicators under mean uncertainty

Mean uncertainty	Reliable indicator interval β	β uncertainty
2%	[1.9565,2.8437]	18%
4%	[1.6162,3.0856]	31%
6%	[1.2859,3.3611]	45%
8%	[0.9019,3.6857]	61%
10%	[0.5351,4.033]	77%
12%	[0.1838,4.4254]	92%

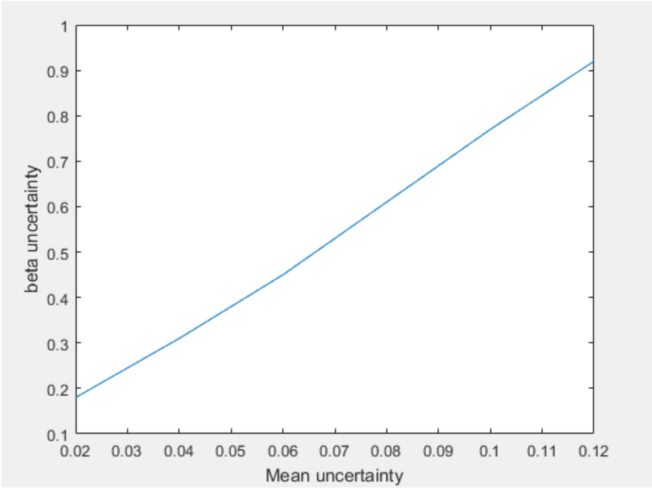

Fig. 4 The relationship between the mean uncertainty and the uncertainty of reliable Index

(1) From Table 4 and Fig. 4, it can be known that as the uncertainty of the mean value increases, the uncertainty of the reliable index also increases, and it shows a proportional relationship between them.

(2) Considering the uncertainty of the standard deviation X and Y of the variables, the reliable index is calculated by the proposed method in this paper. The reliable index interval and its uncertainty are shown in Table 4. The relationship between the standard deviation uncertainty and the uncertainty of the reliable index is shown in Fig. 5.

Table 4 The computational results of reliable indicators under standard deviation uncertainty

Standard deviation uncertainty	Reliable indicator interval β	β uncertainty
5%	[2.1907,2.4617]	5%
10%	[2.1451,2.6333]	10%
15%	[2.1073,2.7390]	13%
20%	[1.9543,2.8797]	19%
25%	[1.8579,3.0693]	25%
30%	[1.7899,3.2813]	29%

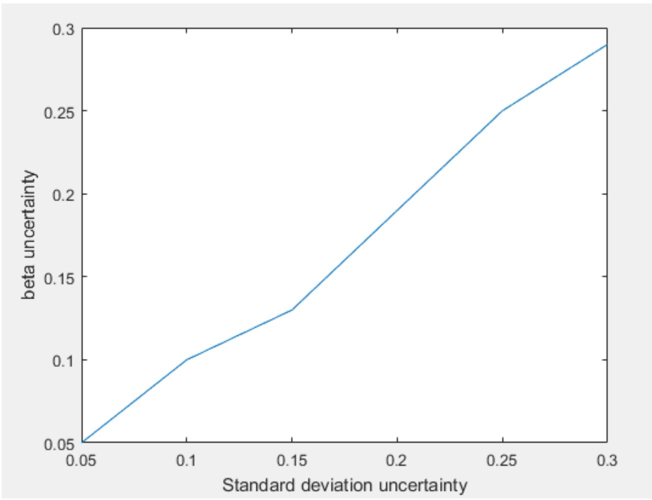

Fig. 5 The relationship between Uncertainty of Standard Deviation and Uncertainty of Reliable Index

Table 5 and Fig. 5 show that as the uncertainty of the standard deviation doubles, the uncertainty of reliable indicators also increases, and it also shows a proportional relationship.

3)At the same time, the mean uncertainty of the variables X and the uncertainty of the standard deviation Y of the variables are considered. The reliable index is calculated by the proposed method in this paper. The reliable index interval and its uncertainty are shown in Table 5. The relationship of index uncertainty is shown in Fig. 6.

Table 5 The computational results of reliable indicators under the uncertainty of mean and standard deviation

Parameter uncertainty	μ_1	μ_2	σ_3	σ_4	Reliable indicator interval β	β uncertainty
3%	[3.88,4.12]	[4.85,5.15]	[2.425,2.575]	[0.776,0.824]	[2.1715,2.6293]	10%
6%	[3.76,4.24]	[4.7,5.3]	[2.35,2.65]	[0.752,0.848]	[1.7960,2.9441]	24%
9%	[3.64,4.36]	[4.55,5.45]	[2.275,2.725]	[0.728,0.872]	[1.4777,3.2373]	37%
12%	[3.52,4.48]	[4.4,5.6]	[2.2,2.8]	[0.704,0.896]	[1.2218,3.5555]	49%
15%	[3.4,4.6]	[4.25,5.75]	[2.125,2.875]	[0.68,0.92]	[0.9298,3.9225]	61%
18%	[3.28,4.72]	[4.1,5.9]	[2.05,2.95]	[0.656,0.944]	[0.6694,4.2472]	73%

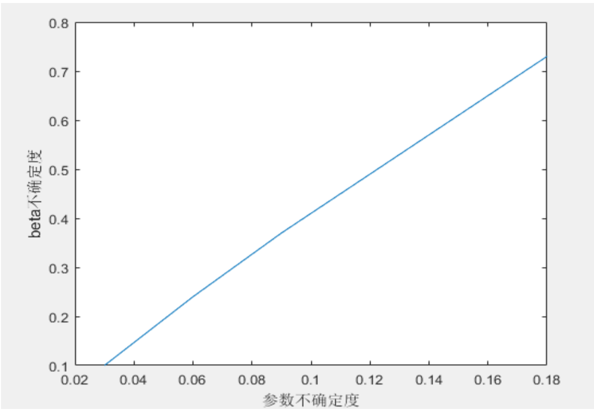

Fig. 6 The relationship between mean and standard deviation uncertainty and reliability index uncertainty

Table 5 and Fig. 6 show that as the uncertainty of the mean and standard deviation doubles, the uncertainty of the reliable indicator also increases, and it also shows a proportional relationship.

3.2 The cantilever beam problem

The structure of a cantilever beam is shown in Fig. 7. The length of the beam is L and the cross-sectional dimensions are t and h. The top of the cantilever beam is acted by the horizontal P_x and vertical forces P_y which are shown in Fig.7. The maximum stress at the fixed end of the cantilever beam cannot exceed the yield strength limit value S. The difference between the allowable stress of the material and the maximum stress of the fixed end of the cantilever beam is selected as the structural function, and then the limit state equation of cantilever beam is obtained as

$$G = S - \frac{6P_xL}{t^2h} - \frac{6P_yL}{th^2}, \tag{18}$$

in which $L = 1m$, $S = 320\,MPa$.

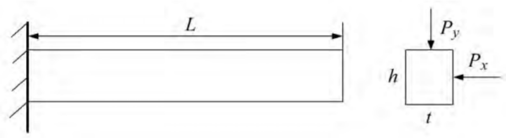

Fig. 7 The Cantilever beam

The important parameters t and h , the horizontal P_x and vertical forces P_y of the cantilever beam are selected as random variables, and they are independent each other. The specific random variable distribution parameters are shown in Table 6.

Table 6 The value of each random variable distribution parameter

Random Variables	Mean	Standard deviation	Distribution type
t (m)	0.1	0.01	Normal
h (m)	0.2	0.015	Normal
$P_x(10^4 N)$	5	0.625	Normal
$P_y(10^4 N)$	2.5	0.15	Normal

Let $x_1 = t, x_2 = h, x_3 = P_x$, $x_4 = P_y$ then the formula (18) will become:

$$G(x) = 32000 - \frac{6x_3}{x_1^2 x_2} - \frac{6x_4}{x_1 x_2^2} \tag{19}$$

For the values in Table 6, we will consider the mean uncertainty of the variables t and h as well as the uncertainty of the standard deviation of P_x and P_y at the same time. The reliability index is calculated by the proposed method in this paper. The obtained reliability index interval and its uncertainty degree are shown in Table 7, and the relationship between the uncertainty of mean value and standard deviation and the uncertainty of reliability index is shown in Fig. 8.

Table 7 The computational results of reliability index under the uncertainty of the mean and standard deviation

Parameter uncertainty	μ_t	μ_h	σ_{p_x}	σ_{p_y}	Reliable index β	β uncertainty
2%	[0.098,0.102]	[0.196,0.204]	[0.6125,0.6325]	[0.147,0.153]	[1.9964,2.6342]	13.77%
4%	[0.096,0.104]	[0.192,0.208]	[0.6,0.65]	[0.144,0.156]	[1.7165,2.7987]	23.97%
6%	[0.094,0.106]	[0.188,0.212]	[0.5875,0.6625]	[0.141,0.159]	[1.4399,3.0362]	35.66%
8%	[0.092,0.108]	[0.184,0.216]	[0.575,0.675]	[0.138,0.162]	[1.1690,3.3208]	47.93%
10%	[0.09,0.11]	[0.18,0.22]	[0.5625,0.6875]	[0.135,0.165]	[0.8946,3.5952]	60.15%
12%	[0.088,0.112]	[0.176,0.224]	[0.55,0.7]	[0.132,0.168]	[0.6118,3.8231]	72.41%

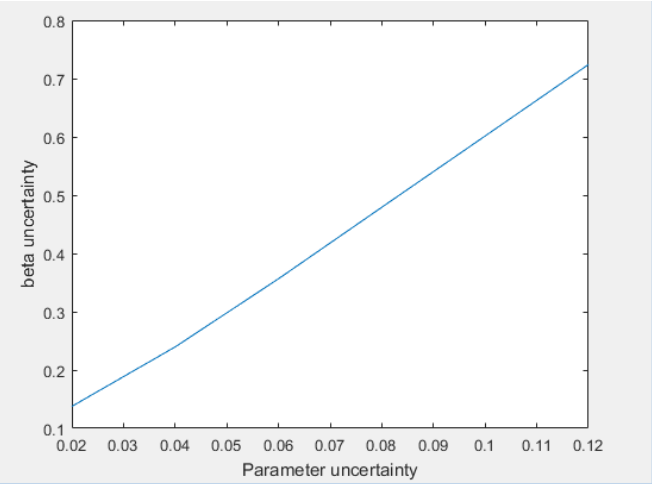

Fig. 8 The relationship between the uncertainty of mean and standard deviation and reliability index uncertainty

From Table 7 and Fig. 8, it can be known that when the uncertainty of the mean and standard deviation is 2%, the uncertainty of the reliability index of the cantilever beam structure is 13.77%. With the doubling of the uncertainty of the mean and the standard deviation, the uncertainty of the reliable index also increases, and it shows a proportional relationship. It also indicates that the parameter uncertainty has a certain effect on the structure reliability.

4. Conclusion

This paper proposes a new structural reliability analysis method based on the simulated annealing algorithm and the external penalty function method. Also, we can draw the following conclusions:

1)Taking into account the uncertainty of the structural parameters, the proposed method in this paper exploits the external penalty function method deal with the constraint conditions, and the simulated annealing method is used to solve the structural reliability analysis problem.

2)The proposed method in this paper can be used to solve the reliability analysis of limit state equation with a certain degree of nonlinearity, the existence of parameter uncertainties and the mixed reliability model of interval variables. The results of numerical examples show that for the limit state equation with a high degree of non-linearity, the simulated annealing external penalty function method can still obtain more accurate and reliable indicators.

3)Numerical results of numerical examples and engineering examples show that parameter uncertainty has a greater impact on the reliability, and there is a positive correlation between the parameter uncertainty and the reliability index uncertainty. In the future research and development, the proposed method in this paper can be extended to the future research considering the influence of the uncertainty with different parameter types and multiple distribution types on the structural reliability.

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