A new correlation reliability analysis method based on evidence theory and ellipsoidal model

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Abstract

In this paper, we propose a new correlation reliability analysis method based on evidence theory and ellipsoidal model. In general, if the distribution of the variables and their distribution parameters are notified, the reliability will be difficult to evaluate. Moreover, in the case of restricted experiment data and confined cognitive level, it is awfully difficult to evaluate the reliability of the structure and calculate the failure probability. The proposed method in this paper combines the evidence theory, the ellipsoidal model and the correlation of variables. We analyze the distribution of BPAs, and also the relationship between limit–state function and failure probability. Finally, numerical simulations demonstrate the stability and effectivity of the proposed method in this paper.

Keywords

Reliability analysis; Evidence theory; Ellipsoidal model; Belief measure; Plausibility measure.

1. Introduction

With different type of uncertain factors, such as material, load, measure, and shape, etc, the actual structure usually has certain risk of failure. Therefore, the failure probability of structure will be pre–estimated before the practical engineering application so as to avoid casualties. Many advanced computational methods are developed to solve lost of engineering problems [1-4]. Dempster [5] and Shafter [6] put forward the evidence theory which is usually called Dempster–Shafer (D–S) theory [7]. Evidence theory, fuzzy set, possibility theory, and convex model can be used to analyze the reliability of the structure when the distribution of variables is unknown. Besides, evidence theory is applicable to dealing with incomplete information, uncertain information and even conflict information. Therefore, the evidence theory and the fuzzy set are usually applied to non–probabilistic reliability analysis of structures. Bai et al [8] contrast four kinds of elementary moment between D–S theory and MCS method. Bai et al [9] analysed R square and the deviation of small failure probability, and the data of the quadratic polynomial, radial basis function (RBF), and High dimensional model representation (HDMR) are compared with each other. Jiang et al [10] added the ellipsoidal model and correlation coefficient to evidence theory, and analyzed the reliability of structure with coefficient. Wang et al [11] used evidence theory and ellipsoid model to analyze the reliability considering the correlation between different variables. Jiang et al [12] transformed the evidence variables into the probability variables by means of homogenization theory, and analyzed the reliability with evidence variables and probability variables. Jiang et al [13] used the evidence theory and the copula function to analyze the reliability considering the correlation between different variables.

This paper adopted the evidence theory and the ellipsoid model, and analyzes the reliability considering the correlation between different variables. Numerical simulations show that not only different interval numbers have the influence on BPAs, but also different correlation coefficients have the influence on BPAs.

2. Evidence theory

If the number of variables is 2 and the frame of discernment (FD) is given as $\boldsymbol{X}=\{X_1,X_2\}$, a power set $2^X=\{\phi,X_1,X_2,\{X_1,X_2\}\}$ will be contrasted by the possible subset propositions of $\boldsymbol{X}=\{X_1,X_2\}$. Besides, the limit–state function (LSF) is $g(X)$.

The interval of X_1 and X_2 is divided into several intervals, and any two intervals can form a focal element d_k. The basic probability assignment (BPA) corresponding to every focal element d_k can be calculated by the mass function $m:2^X\to[0,1]$. Besides, the mass function which satisfies several conditions is shown as follows

$$m(\phi)=0 \tag{1}$$

$$\sum_{A\in X} m(\phi)=1 \tag{2}$$

The Cartesian product D is defined as follows

$$D\in A_1\times A_2\cdots A_n=\{d_k=\{a_1,a_2\cdots a_n\},a_j\in A_j,j=1,2\cdots n\} \tag{3}$$

If the information is not conflicting, the BPA can be given as follows

$$m(d_k)=\begin{cases}\prod_{j=1}^{n} m(a_j) & \{d_k=\{a_1,a_2\cdots a_n\},j=1,2\cdots n\}\\ 0 & otherwise\end{cases} \tag{4}$$

If the information is conflicting, for two BPAs $m_1(B)$ and $m_2(C)$, the BPA can be obtained as follows

$$m(A)=\begin{cases}\dfrac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-K} & A\neq\phi\\ 0 & A=\phi\end{cases} \tag{5}$$

$$K=\sum_{B\cap C=\phi} m_1(B)m_2(C)<1 \tag{6}$$

where K represents the grade of conflict between the two evidence. The larger K, the greater the conflict between the evidences. Besides, $(1-K)^{-1}$ can prevent the measure of nonzero belief from being assigned to the empty set.

The belief measure Bel and plausibility measure Pl can be obtained as follows

$$\mathrm{Bel}(A)=\sum_{B\subset A} m(B),A\in X \tag{7}$$

$$\mathrm{Pl}(A)=\sum_{B\cap A=\phi} m(B)=1-\mathrm{Bel}(\overline{A}),\ \ A\in X,\overline{A}=\{Y\mid Y\in X \text{ and } Y\notin A\} \tag{8}$$

If the safety region is G, the interval of failure probability is shown as follows

$$\mathrm{Bel}(G)\le P_{\mathrm{f}}\le \mathrm{Pl}(G) \tag{9}$$

If the failure region is $\Omega_{\mathrm{F}}=\{X\mid g(X)\le 0\}$, the failure probability is shown as follows

$$P_{\mathrm{f}}=\Pr\{X\subset\Omega_{\mathrm{F}}\} \tag{10}$$

According to the LSF, the interval of LSF in every focal element d_k can be obtained s follows

$$[g_{\min},g_{\max}]=\left[\min_{X\in d_k} g(X),\max_{X\in d_k} g(X)\right] \tag{11}$$

If $g_{\min}>0$ and $g_{\max}>0$, it indicates that $D\subseteq G$ and the focal element will contribute to both $\mathrm{Bel}(G)$ and $\mathrm{Pl}(G)$; if $g_{\min}<0$ and $g_{\max}<0$, it indicates that the focal element will contribute to neither $\mathrm{Bel}(G)$ nor $\mathrm{Pl}(G)$; if $g_{\min}<0$ and $g_{\max}>0$, it indicates the focal element will only contribute to $\mathrm{Pl}(G)$.

3. Ellipsoidal model

If the evidence theory variables $X_i \in \boldsymbol{X}_i^{\mathrm{I}}=[X_i^{\mathrm{L}}, X_i^{\mathrm{R}}]$, ellipsoidal center, ellipsoidal radius and ellipsoidal variance can be obtained as follows

$$\boldsymbol{X}^{\mathrm{C}}=\frac{\boldsymbol{X}^{\mathrm{R}}+\boldsymbol{X}^{\mathrm{L}}}{2} \tag{12}$$

$$\boldsymbol{X}^{\mathrm{W}}=\frac{\boldsymbol{X}^{\mathrm{R}}-\boldsymbol{X}^{\mathrm{L}}}{2} \tag{13}$$

$$\boldsymbol{D}(\boldsymbol{X}^{\mathrm{I}})=(\boldsymbol{X}^{\mathrm{W}})^2 \tag{14}$$

On the basis of the relevance matrix $\boldsymbol{\rho}$, the covariance matrix can be obtained as follows

$$\boldsymbol{C}(\boldsymbol{X})=\boldsymbol{\rho}\times\sqrt{\boldsymbol{D}^{\mathrm{T}}(\boldsymbol{X}^{\mathrm{I}})\times\boldsymbol{D}(\boldsymbol{X}^{\mathrm{I}})} \tag{15}$$

Then, the ellipsoidal function can be obtained as follows

$$G(\boldsymbol{X})=(\boldsymbol{X}-\boldsymbol{X}^{\mathrm{C}})^{\mathrm{T}}\boldsymbol{C}^{-1}(\boldsymbol{X}-\boldsymbol{X}^{\mathrm{C}})\le 1 \tag{16}$$

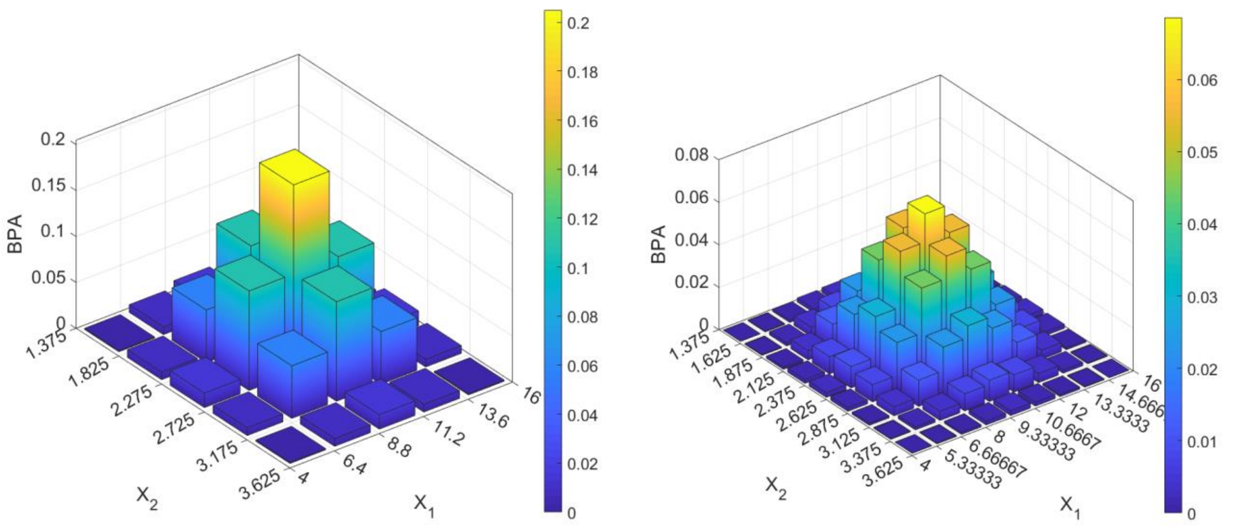
If the value of ellipsoidal function corresponding to four node of focal element are all positive, the BPAs corresponding to the focal element will be set to zero. Besides, in order to keep the sum of all BPAs, the remaining BPAs need to be divided by the sum of all the BPAs to make sure that the sum of all the BPAs is 100%.

4. Numerical experiment 1

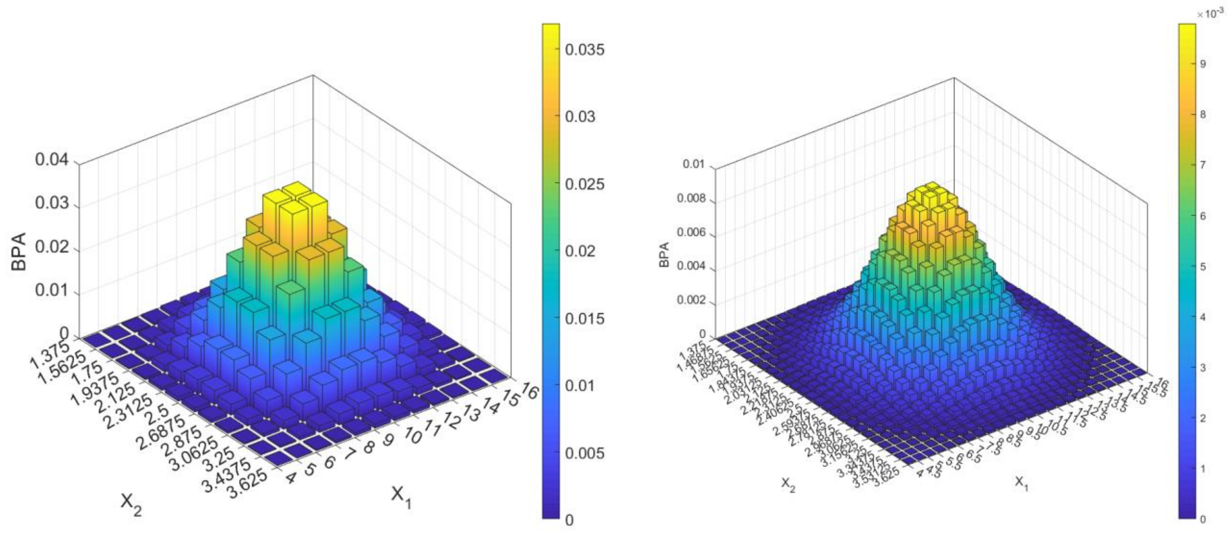
Suppose that the variable $\boldsymbol{X}=[X_1, X_2]$ obeys the normal distribution. The frame of discernments of X_1 and X_2 are $[4,16]$ and $[1.375, 3.625]$, respectively. The limit–state function is given as

$$g(\boldsymbol{X})=18.46-7.48X_1X_2^{-3} \tag{17}$$

With different predetermined number of focal element, the distribution of BPAs will vary conspicuously. In order to observe the vary of the distribution of BPAs, the interval of X_1 and X_2 is divided into 5, 9, 12 and 24 intervals, and the corresponding distribution of BPAs are shown as Fig. 1.



(a) 5 intervals (b) 9 intervals



(c) 12 intervals (d) 24 intervals

Fig. 1 The distribution of BPAs

We can draw some conclusion from Fig. 1:

(i) The more focal element, the less BPAs.

(ii) The more focal element, the distribution of BPAs will be more smooth.

If the interval of X_1 and X_2 is divided into 15 intervals, the corresponding BPA of every focal elements are shown as Table 1.

Table 1 The marginal BPAs

X_1^{low}	X_1^{up}	BPA	X_2^{low}	X_2^{up}	BPA
4.000	4.800	0.003 320	1.375	1.525	0.003 320
4.800	5.600	0.009 267	1.525	1.675	0.009 267
5.600	6.400	0.022 087	1.675	1.825	0.022 087
6.400	7.200	0.044 948	1.825	1.975	0.044 948
7.200	8.000	0.078 109	1.975	2.125	0.078 109
8.000	8.800	0.115 911	2.125	2.275	0.115 911
8.800	9.600	0.146 884	2.275	2.425	0.146 884
9.600	10.400	0.158 949	2.425	2.575	0.158 949
10.400	11.200	0.146 884	2.575	2.725	0.146 884
11.200	12.000	0.115 911	2.725	2.875	0.115 911
12.000	12.800	0.078 109	2.875	3.025	0.078 109
12.800	13.600	0.044 948	3.025	3.175	0.044 948
13.600	14.400	0.022 087	3.175	3.325	0.022 087
14.400	15.200	0.009 267	3.325	3.475	0.009 267
15.200	16.000	0.003 320	3.475	3.625	0.003 320

According to the BPA of every focal element shown as above, we can obtain the distribution of BPAs as Fig. 2. Then, according to the BPAs of focal element, we can obtain the value of belief measure Bel and plausibility measure Pl. The curves of Bel, Pl and P_f are shown as Fig. 3.

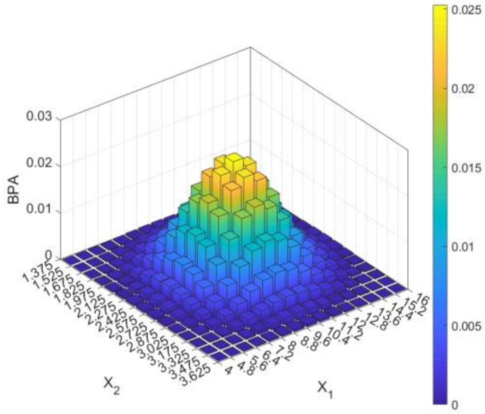


Fig. 2 The distribution of BPAs

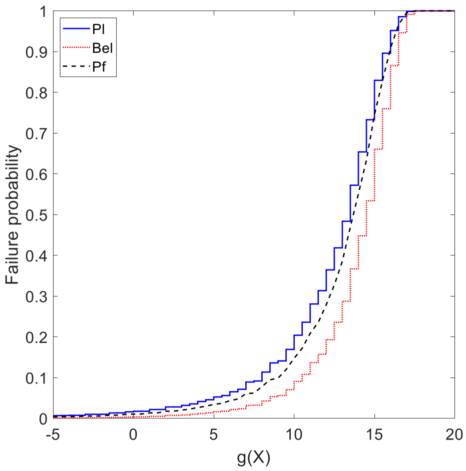


Fig. 3 The contrast among Bel, Pl and P_f

We can draw some conclusion from Fig. 3:

(i) If the structure fails when $g(\boldsymbol{X})$ is less than -5, the failure probability of the structure is very small.

(ii) If the structure fails when $g(\boldsymbol{X})$ is less than 17, the failure probability of the structure would be closed to 100%.

5. Numerical experiment 2

Suppose that the variable $\boldsymbol{X}=[X_1,X_2]$ obeys the normal distribution. The frame of discernments of X_1 and X_2 are both $[3.6,8.4]$. The limit–state function is shown as follows

$$g(\boldsymbol{X})=-0.3X_1^2X_2-X_2+0.8X_1+1 \quad (18)$$

If the interval of X_1 and X_2 is divided into 15 intervals, the corresponding BPA of every focal elements are shown as Table 2.

Table 2 The marginal BPAs

X^{low}	X^{up}	BPA	X^{low}	X^{up}	BPA
3.600	3.920	0.003 320	6.160	6.480	0.146 884
3.920	4.240	0.009 267	6.480	6.800	0.115 911
4.240	4.560	0.022 087	6.800	7.120	0.078 109
4.560	4.880	0.044 948	7.120	7.440	0.044 948
4.880	5.200	0.078 109	7.440	7.760	0.022 087
5.200	5.520	0.115 911	7.760	8.080	0.009 267

5.520	5.840	0.146 884	8.080	8.400	0.003 320
5.840	6.160	0.158 949			

According to the BPA of every focal element shown as above, we can obtain the distribution of BPAs as Fig. 4. Then, according to the BPAs of focal element, we can obtain the value of belief measure Bel and plausibility measure Pl. The curves of Bel, Pl and P_f are shown as Fig. 5.

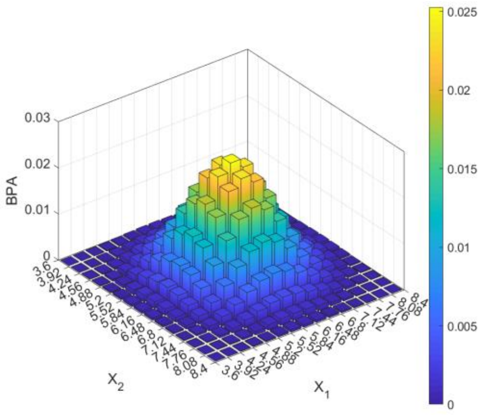


Fig. 4 The distribution of BPAs

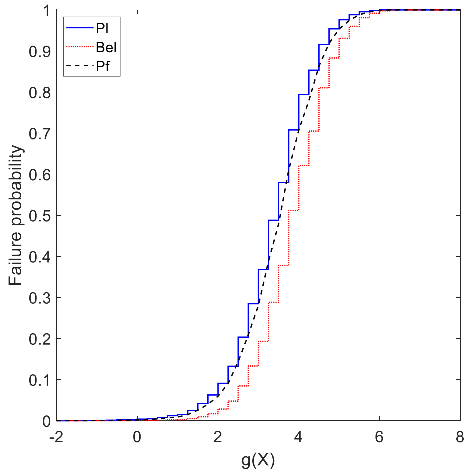


Fig. 5 The contrast among Bel, Pl and P_f

We can draw some conclusion from Fig. 5:

(i) If the structure fails when $g(\boldsymbol{X})$ is less than 0, the failure probability of the structure is very small.

(ii) If the structure fails when $g(\boldsymbol{X})$ is less than 6, the failure probability of the structure would be closed to 100%.

Herein, different correlation coefficients are chosen, and the corresponding ellipsoidal model are shown as Fig. 6.

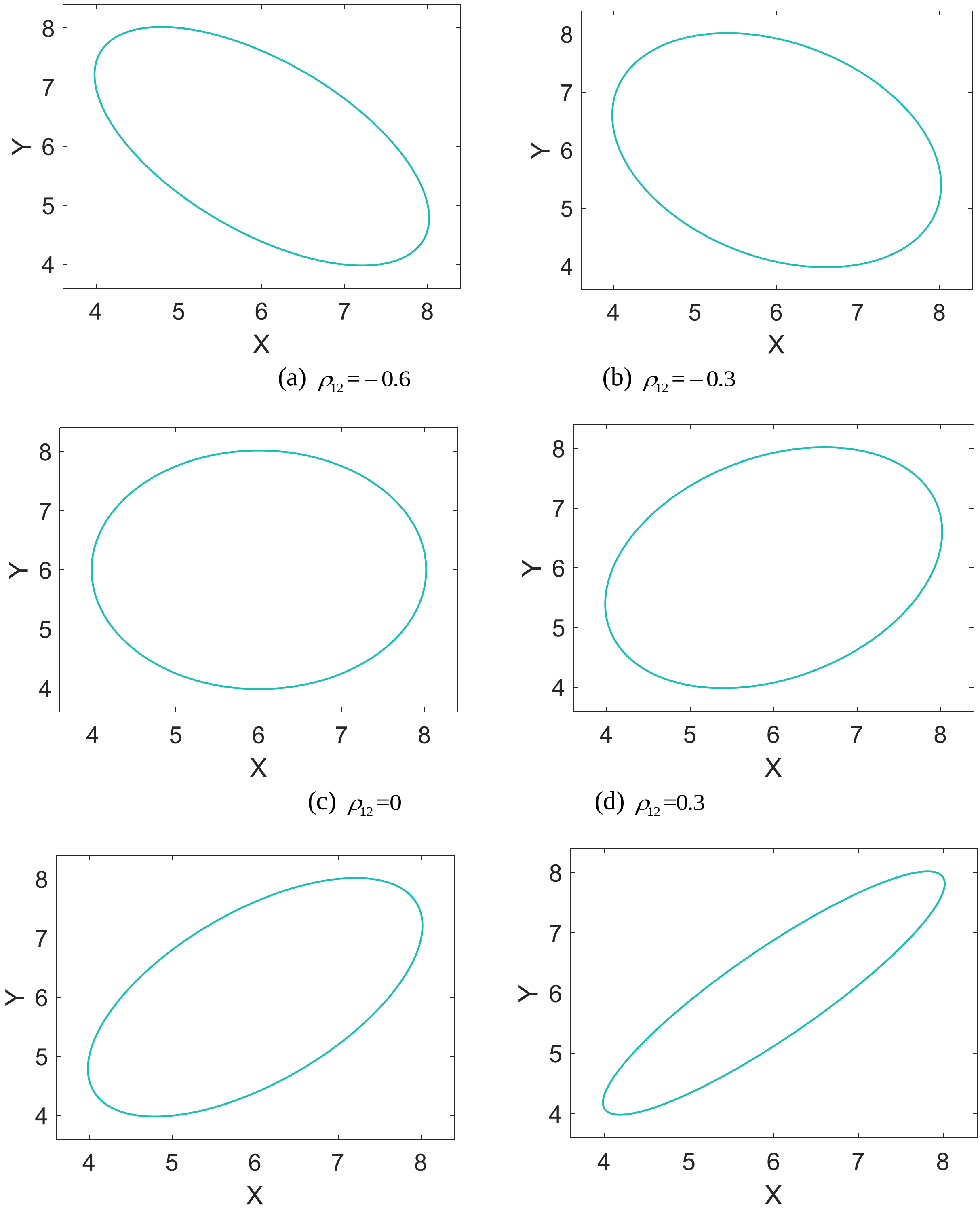
Some conclusion can be drawn from Fig. 6:

(i) The larger the correlation coefficient, the smaller the area of the ellipsoid.

(ii) If the correlation coefficient is set to 0, the long axis of the ellipsoid is horizontal.

(iii) If the sign of correlation coefficient is opposite, the axis of the ellipsoid model will be shift at the same time.

Herein, the correlation coefficient is set to 0.6 and the number of intervals is set to 6, 8, 10 and 15, respectively. The corresponding distribution of BPAs is shown as Fig. 7.

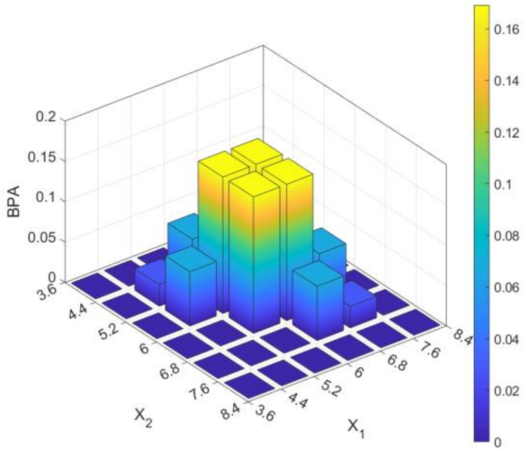


(a) $\rho_{12}=-0.6$ (b) $\rho_{12}=-0.3$

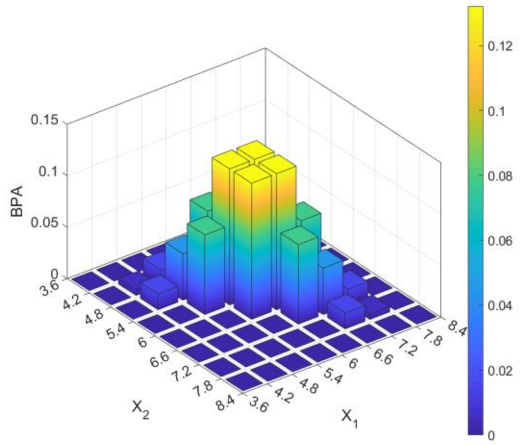
(c) $\rho_{12}=0$ (d) $\rho_{12}=0.3$

(e) $\rho_{12}=0.6$ (f) $\rho_{12}=0.9$

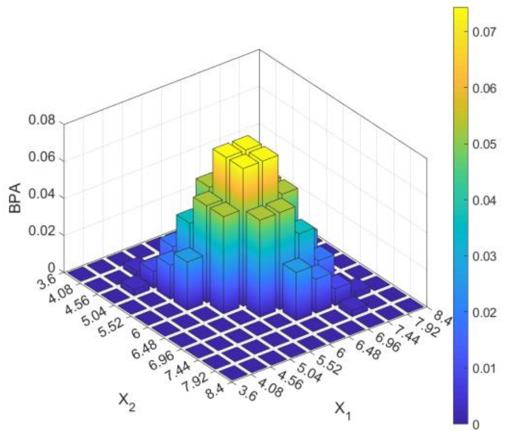
Fig. 6 The ellipsoidal model with different correlation coefficient



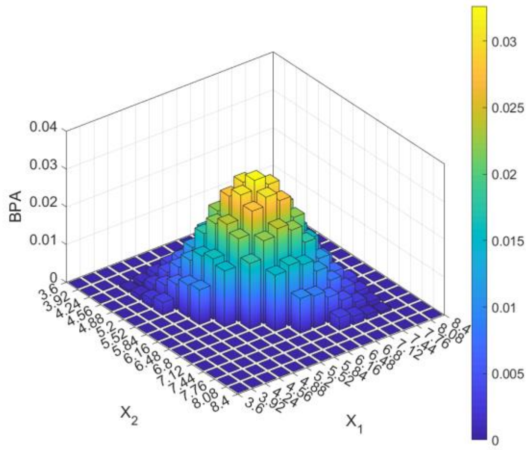
(a) 6 intervals



(b) 8 intervals



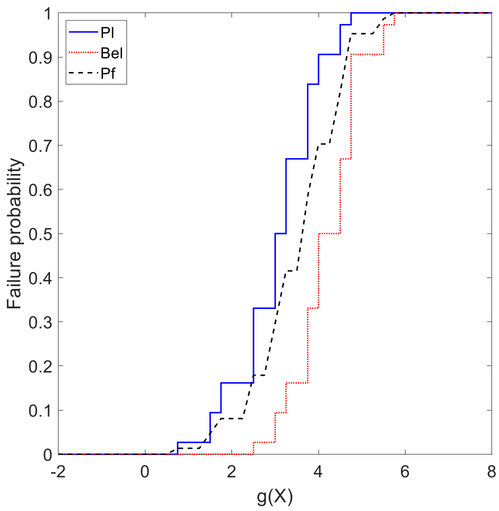
(c) 10 intervals



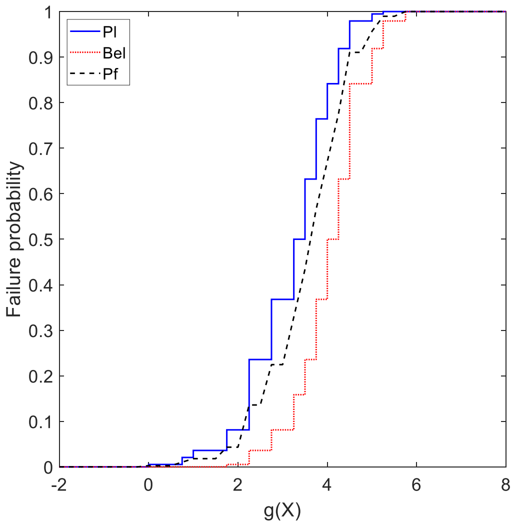
(d) 15 intervals

Fig. 7 The distribution of BPAs

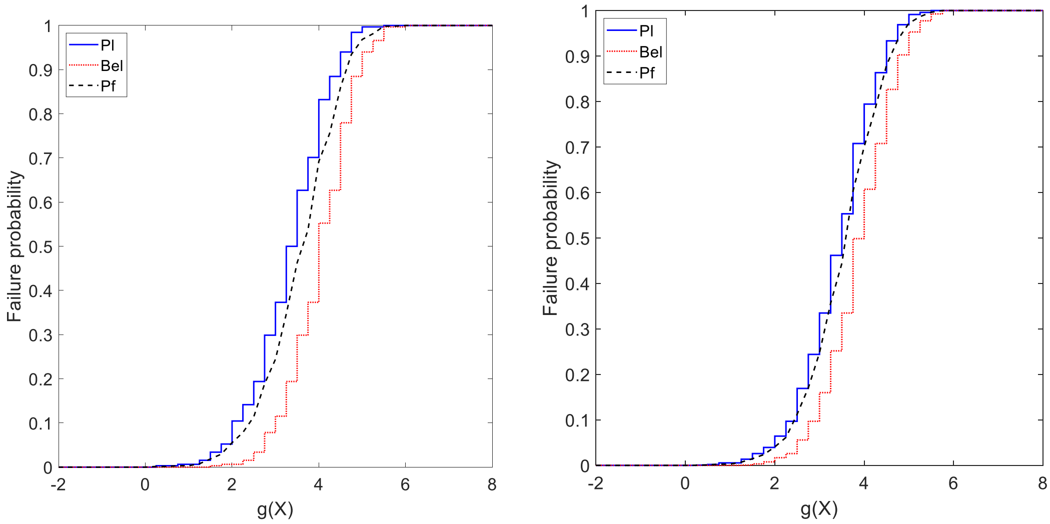
Then, according to the BPAs of focal element, we can obtain the value of belief measure Bel and plausibility measure Pl. The curve of Bel, Pl and P_f is shown as Fig. 8.



(a) 6 intervals



(b) 8 intervals



(c) 10 intervals (d) 15 intervals

Fig. 8 The contrast among Bel, Pl and P_f

As can be seen from Fig. 8, the larger the number of intervals, the smoother the curve of Bel, Pl and P_f. As we can see from Fig. 5 and Fig. 8, if we take the correlation into account, the point where the probability of failure is close to zero will change from 0 to 1.

6. Conclusion

A new correlation reliability analysis method based on evidence theory and ellipsoidal model is proposed in this paper. Firstly, the evidence theory is used to analyses the reliability of structure, and the ellipsoidal model is used to analyze the reliability considering the coefficient between different variables. Numerical simulations validate the stability of the proposed algorithm. Furthermore, some conclusion can be drawn:

(i) The larger the correlation coefficient, the smaller the ellipsoid.

(ii) The larger the number of intervals, the smoother the curves of Bel, Pl and P_f.

(iii) If we take into account the correlation, the point where the probability of failure is close to zero will move right.

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