Reliability analysis based on adaptive chaos control method

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Abstract

Regarding the structural reliability analysis, the Hasofer-Lind Rackwitz-Fiessler (HL-RF) algorithm is widely used in the calculation of first-order second-moment method because of its easy implementation and high efficiency. However, when the highly nonlinear examples are solved, the algorithm will appear the phenomenon of oscillation or chaos. In order to improve the convergence performance of the HL-RF algorithm, the adaptive chaos control method is used to improve the HL-RF algorithm and applied to the reliability analysis. In this paper, eight algorithms are used to improve the HL-RF algorithm, and the comparison was made with the Latin Hypercube Monte Carlo Simulation (LHS) method. The results show that: when the reliable index of the structure is large, the Latin hypercube Monte Carlo method is difficult to solve, and the practicability is poor. When the nonlinear degree of the example is high, the convergence of other algorithms is poor, but both the hybrid chaotic control method and the adaptive chaotic control method show excellent performances. In addition, five time-varying reliability models are solved by using adaptive chaos control method and compared with each other. Therefore, the improved HL-RF algorithm based on the adaptive chaos control method has high application value in practical engineering.

Keywords

Reliability analysis; HL-RF method; Chaos Control Method; Monte Carlo Method; Time-varying reliability.

1. Introduction

Reliability analysis is mainly used to evaluate the failure risk of structures, so as to avoid the casualties caused by the structural failure. Many experts develop some advanced methods which are applied to engineering problems [1-4]. Also, because the HL-RF method is simple and efficient, it is usually nested inside the first-order reliability method (FORM). When the method is applied to highly nonlinear function, the convergence of the algorithm is poor, so the scope of application is limited. Therefore, many scholars have studied this algorithm. Due to the poor robustness of Mean Value First Order Second Moment (MVFOSM), Wu et al. [5] proposed a more robust Advanced Mean Value (AMV) method. Since AMV has a good convergence for convex functions and a poor convergence for concave functions, Youn et al. [6] proposed a Conjugate Mean Value (CMV) method with better convergence for concave functions. Besides, the Hybrid Mean Value (HMV) method combining the advantages of the AMV method and CMV method is proposed at the same time. Since the convergence of HMV method is poor in solving highly nonlinear problems, Youn et al. [7] proposed an Enhanced Hybrid Mean Value (HMV+) method. In order to improve the convergence speed of the algorithm and overcome the bifurcation phenomenon of the algorithm, Ezzati et al. [8] proposed Conjugate Gradient Analysis (CGA) method. In order to improve the computational efficiency and robustness of the algorithm, Keshtegar et al. [9] proposed Relaxed Mean Value (RMV) method. Yang [10] added Chaos feedback control method to the Stability Transformation Method (STM) [11] and proposed Chaos Control (CC) method. Due to the short step size of CC method and low efficiency, Meng et al. [12, 13] proposed an efficient Modified Chaos Control (MCC) method and a Hybrid

Chaos Control (HCC) method with less function computation. Since the step size has a great influence on the algorithm, Li et al. [14] proposed Adaptive Chaos control method (Adaptive Chaos control, ACC). Hao et al. [15] improved ACC method through the chaotic dynamics theory and proposed Enhanced Chaos Control (ECC) method with faster convergence rate. Kang et al. [16] judged whether the algorithm was convergent by the angle of negative gradient direction, and added a correction coefficient to the CMV method to improve the robustness of the algorithm. Li et al. [17] proposed an improved adaptive chaos control method considering that the decreasing coefficient is beneficial to the convergence of the algorithm.

In this paper, AMV method, CMV method, HMV method, RMV method, CC method, MCC method, HCC method, and ACC method were used to improve HL-RF method, and their detailed comparison was listed. The results show that ACC method has better convergence and need little iteration. In addition, ACC method is used to analyze the uncertainty of standard deviation, the correlation between variables and the time-varying reliability of structures.

The Latin Hyper Cube Monte Carlo method is not only computationally intensive, but also difficult to solve the example with too large reliability index. The convergence and computational efficiency of HL-RF method improved by ACC method are better than other algorithms. Therefore, the improved HL-RF method based on the ACC method has less computation and higher application value for practical engineering.

2. The iterative formula of HL-RF method

The transformation of random variables from X-space to U-space is shown as follows [18]

$$\boldsymbol{U} = \frac{\boldsymbol{X} - \boldsymbol{\mu}_X}{\boldsymbol{\sigma}_X} \tag{1}$$

where $\boldsymbol{\mu}_X$ and $\boldsymbol{\sigma}_X$ is mean value and standard deviation of normal variable $\boldsymbol{X}$, respectively; $\boldsymbol{U}$ is the variable that obeys the standard normal distribution.

The updated formula for the sensitivity of the limit state equation is given as [19]

$$\boldsymbol{\alpha}^{(k+1)} = -\frac{\nabla^{\mathrm{T}} g(\boldsymbol{U}^{(k)})}{\left\|\nabla^{\mathrm{T}} g(\boldsymbol{U}^{(k)})\right\|} \tag{2}$$

The updating formula of the reliable index is as follows [19]

$$\beta^{(k+1)} = \frac{g(\boldsymbol{U}^{(k)}) - \nabla^{\mathrm{T}} g(\boldsymbol{U}^{(k)})\boldsymbol{U}^{(k)}}{\left\|\nabla^{\mathrm{T}} g(\boldsymbol{U}^{(k)})\right\|} \tag{3}$$

3. The update formula for MPP points

(i) If the limit-state function is a convex function, the AMV method has better convergence. Besides, the update formula is shown as follows

$$\begin{cases} \boldsymbol{U}_{\mathrm{AMV}}^{(k)} = \boldsymbol{0} & k = 1 \\ \boldsymbol{U}_{\mathrm{AMV}}^{(k+1)} = \beta^{(k+1)}\boldsymbol{\alpha}^{(k+1)} & k > 1 \end{cases} \tag{4}$$

(ii) If the limit-state function is a concave function, the CMV method has better convergence. Besides, the update formula is shown as follows

$$\begin{cases} \boldsymbol{U}_{\mathrm{CMV}}^{(k)} = \boldsymbol{0} & k = 0 \\ \boldsymbol{U}_{\mathrm{CMV}}^{(k)} = \boldsymbol{U}_{\mathrm{AMV}}^{(k)} & k = 1,2 \\ \boldsymbol{U}_{\mathrm{CMV}}^{(k+1)} = \beta^{(k+1)} \dfrac{\boldsymbol{\alpha}^{(k+1)} + \boldsymbol{\alpha}^{(k)} + \boldsymbol{\alpha}^{(k-1)}}{\left\|\boldsymbol{\alpha}^{(k+1)} + \boldsymbol{\alpha}^{(k)} + \boldsymbol{\alpha}^{(k-1)}\right\|} & k \ge 2 \end{cases} \tag{5}$$

(iii) Since the convergence of AMV and CMV is complementary, the HMV method is proposed in Ref. [6].

$$\begin{cases} \boldsymbol{U}_{\text{HMV}}^{(k+1)} = \boldsymbol{U}_{\text{AMV}}^{(k+1)} & \left(\boldsymbol{\alpha}^{(k+1)} - \boldsymbol{\alpha}^{(k)}\right)\left(\boldsymbol{\alpha}^{(k)} - \boldsymbol{\alpha}^{(k-1)}\right) > 0 \\ \boldsymbol{U}_{\text{HMV}}^{(k+1)} = \boldsymbol{U}_{\text{CMV}}^{(k+1)} & \left(\boldsymbol{\alpha}^{(k+1)} - \boldsymbol{\alpha}^{(k)}\right)\left(\boldsymbol{\alpha}^{(k)} - \boldsymbol{\alpha}^{(k-1)}\right) \le 0 \end{cases} \tag{6}$$

(iv) In order to improve the robustness of the algorithm, the RMV method was proposed in Ref. [9]. The update formula is given as

$$\boldsymbol{U}_{\text{RMV}}^{(k+1)} = \beta^{(k+1)} \frac{\boldsymbol{U}_{\text{RMV}}^{\prime(k+1)}}{\left\|\boldsymbol{U}_{\text{RMV}}^{\prime(k+1)}\right\|} \tag{7}$$

$$\boldsymbol{U}_{\text{RMV}}^{\prime(k+1)} = \left(1-\alpha^{(k)}\right)\boldsymbol{U}_{\text{RMV}}^{\prime(k+1)} + \alpha^{(k)}\boldsymbol{U}_{\text{AMV}}^{(k+1)} \tag{8}$$

$$\begin{cases} \alpha^{(k+1)} = \alpha_{\max} & \boldsymbol{U}_{\text{RMV}}^{\prime(k+1)} - \boldsymbol{U}_{\text{RMV}}^{(k)} \ge \boldsymbol{U}_{\text{RMV}}^{(k)} - \boldsymbol{U}_{\text{RMV}}^{(k+1)} \\ \alpha^{(k+1)} = \dfrac{\alpha_{\max}}{C^m}, m = m+1 & \boldsymbol{U}_{\text{RMV}}^{\prime(k+1)} - \boldsymbol{U}_{\text{RMV}}^{(k)} < \boldsymbol{U}_{\text{RMV}}^{(k)} - \boldsymbol{U}_{\text{RMV}}^{(k+1)} \end{cases} \tag{9}$$

$$\alpha_{\max} = \min(\alpha^{(1)}, \beta^{(k+1)}) \tag{10}$$

where the initial value of m is 2; $\alpha^{(1)}$ =2, C =1.05. In general, the range of C is $[1.005,1.1]$.

(v) Due to the poor convergence of HMV method in solving nonlinear problems, the CC method was proposed in Ref. [10]. The update formula is shown as follows

$$\boldsymbol{U}_{\text{CC}}^{(k+1)} = \boldsymbol{U}_{\text{CC}}^{(k)} + \lambda \boldsymbol{C}\left(\boldsymbol{U}_{\text{AMV}}^{(k+1)} - \boldsymbol{U}_{\text{CC}}^{(k)}\right), \quad 0 < \lambda < 1 \tag{11}$$

(vi) Since the step size of CC method is too short and the CC method is inefficient, the MCC method is proposed in Ref. [12]. The update formula is shown as follows

$$\boldsymbol{U}_{\text{MCC}}^{(k+1)} = \beta^{k+1} \frac{\boldsymbol{U}_{\text{CC}}^{(k+1)}}{\left\|\boldsymbol{U}_{\text{CC}}^{(k+1)}\right\|} \tag{12}$$

(vii) In order to reduce the amount of function calculation, the HCC method was proposed in Ref. [8]. The update formula is shown as follows

$$\begin{cases} \boldsymbol{U}_{\text{HCC}}^{(k+1)} = \boldsymbol{U}_{\text{AMV}}^{(k+1)} & \left(\boldsymbol{\alpha}^{(k+1)} - \boldsymbol{\alpha}^{(k)}\right)\left(\boldsymbol{\alpha}^{(k)} - \boldsymbol{\alpha}^{(k-1)}\right) > 0 \; or \; k < 3 \\ \boldsymbol{U}_{\text{HCC}}^{(k+1)} = \boldsymbol{U}_{\text{MCC}}^{(k+1)} & otherwise \end{cases} \tag{13}$$

(viii) If the value of λ is too large, the convergence of the algorithm is poor; if the value of λ is too small, the efficiency of the algorithm is too low. Therefore, Ref. [14] proposed the ACC method. The update formula is shown as follows

$$\begin{cases} \boldsymbol{U}_{\text{ACC}}^{(k+1)} = \boldsymbol{U}_{\text{AMV}}^{(k+1)} & \left(\boldsymbol{\alpha}^{(k+1)} - \boldsymbol{\alpha}^{(k)}\right)\left(\boldsymbol{\alpha}^{(k)} - \boldsymbol{\alpha}^{(k-1)}\right) > 0 \\ \boldsymbol{U}_{\text{ACC}}^{(k+1)} = \beta^{k+1} \dfrac{\boldsymbol{U}_{\text{ACC}}^{\prime(k+1)}}{\left\|\boldsymbol{U}_{\text{ACC}}^{\prime(k+1)}\right\|} & \left(\boldsymbol{\alpha}^{(k+1)} - \boldsymbol{\alpha}^{(k)}\right)\left(\boldsymbol{\alpha}^{(k)} - \boldsymbol{\alpha}^{(k-1)}\right) \le 0 \end{cases} \tag{14}$$

$$\boldsymbol{U}_{\text{ACC}}^{\prime(k+1)} = \boldsymbol{U}_{\text{ACC}}^{(k)} + \lambda \boldsymbol{C}\left(\boldsymbol{U}_{\text{AMV}}^{(k+1)} - \boldsymbol{U}_{\text{ACC}}^{(k)}\right) \tag{15}$$

$$\lambda = \begin{cases} 0.2\lambda & 0.2\theta^{(k)} > \theta^{(k-1)} \\ \lambda\theta^{(k-1)}/\theta^{(k)} & \theta^{(k)} > \theta^{(k-1)} \ge 0.2\theta^{(k)} \\ \lambda & \theta^{(k)} \le \theta^{(k-1)} \end{cases} \tag{16}$$

$$\theta^{(k)} = \arccos\left(\frac{\boldsymbol{U}_{\text{ACC}}^{\text{T}(k+1)}\boldsymbol{U}_{\text{ACC}}^{(k)}}{\left\|\boldsymbol{U}_{\text{ACC}}^{\text{T}(k+1)}\right\|\left\|\boldsymbol{U}_{\text{ACC}}^{(k)}\right\|}\right) \tag{17}$$

4. Concave function

Assume the variables obey the normal distribution $X_1 \sim N(1.3, 0.55^2)$, $X_1 \sim N(1.0, 0.55^2)$, and the LSF is shown as follows

$$g(\boldsymbol{X}) = -0.3X_1^2X_2 - X_2 + 0.8X_1 + 1 \tag{18}$$

When the simulation number of LHS method is 1×10^5, the obtained reliability indexes are 1.8082, 1.8221 and 1.8110, and the mean value is 1.8138.

If the parameters of RMV method are set according to the recommended value, RMV algorithm does not converge, and the iterative process is shown as Fig. 1.

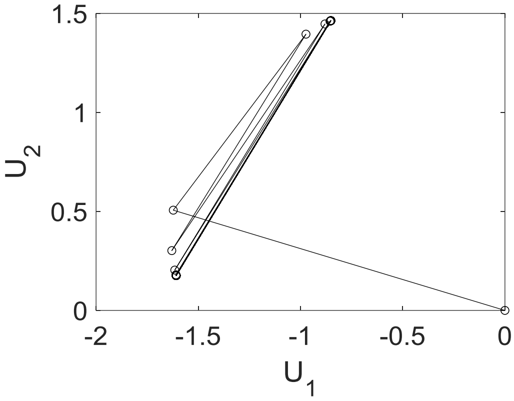


Fig. 1 The oscillation of RMV method

Practical experience shows that the convergence of RMV method is irrelevant to C and m. However, if the value of $\alpha^{(1)}$ is 1, the RMV method will converge.

Herein, we choose $\boldsymbol{C}=[1,0;0,1]$, the initial value of λ is set to 0.3. When the convergence condition is set to $\|\boldsymbol{X}_k - \boldsymbol{X}_{k-1}\|/\|\boldsymbol{X}_{k-1}\| < 1\times10^6$, the MPP point is (0.5264,1.5500), and the reliability index is 1.7258. The number of iterations and the process of iteration are shown as Table 1 and Fig. 2 respectively.

Table 1 The number of iteration

Method	AMV	CMV	HMV	RMV	CC	MCC	HCC	ACC
Iteration	16 times	17 times	17 times	16 times	37 times	25 times	19 times	16 times

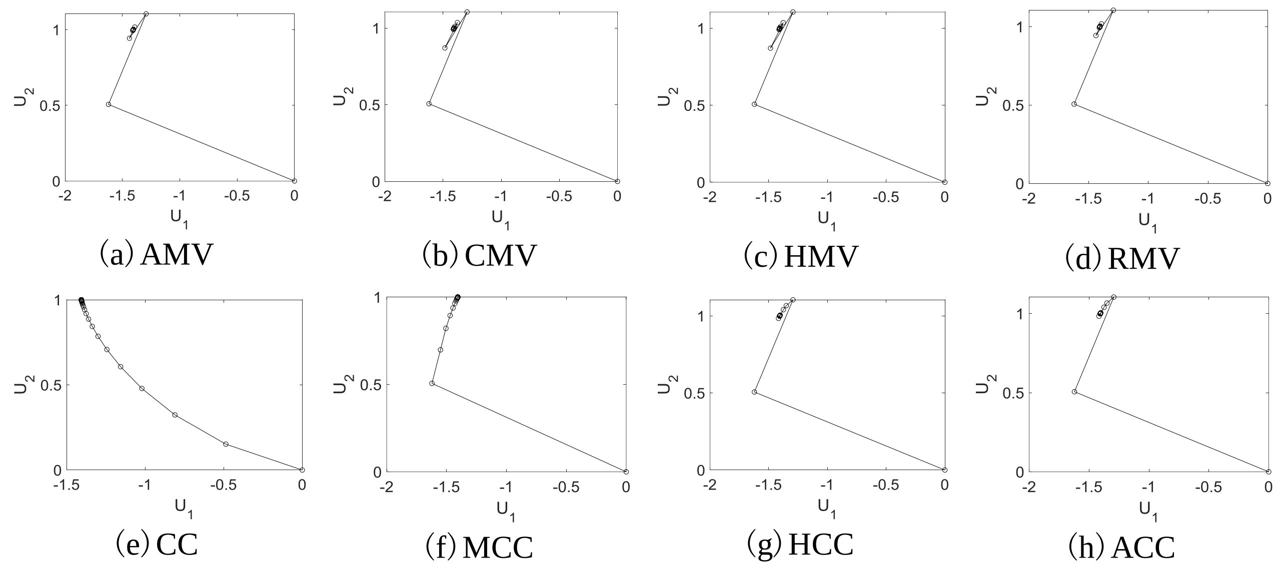


Fig. 2 The process of iteration

Some conclusions can be drawn from Table 1 and Fig. 2:

(i) The number of iterations and process of iterations of AMV method, RMV method, HCC method, ACC method are the same.

(ii) Although both CMV method and HMV method are modified on the basis of AMV method, the computational efficiency of the algorithm is poor.

(iii) The iteration curve of CC method is very smooth, but it's number of iteration is too large. However, the efficiency of MCC method is greatly improved and the computational efficiency is higher.

(iv) If the initial value of λ is changed from 0.3 to 1, the iteration curve of CC method and MCC method will the same as HCC method and ACC method.

5. Convex function

Assume the variables obey the normal distribution: $X_1, X_2 \sim N(6.0, 0.8^2)$, and the LSF is shown as follows

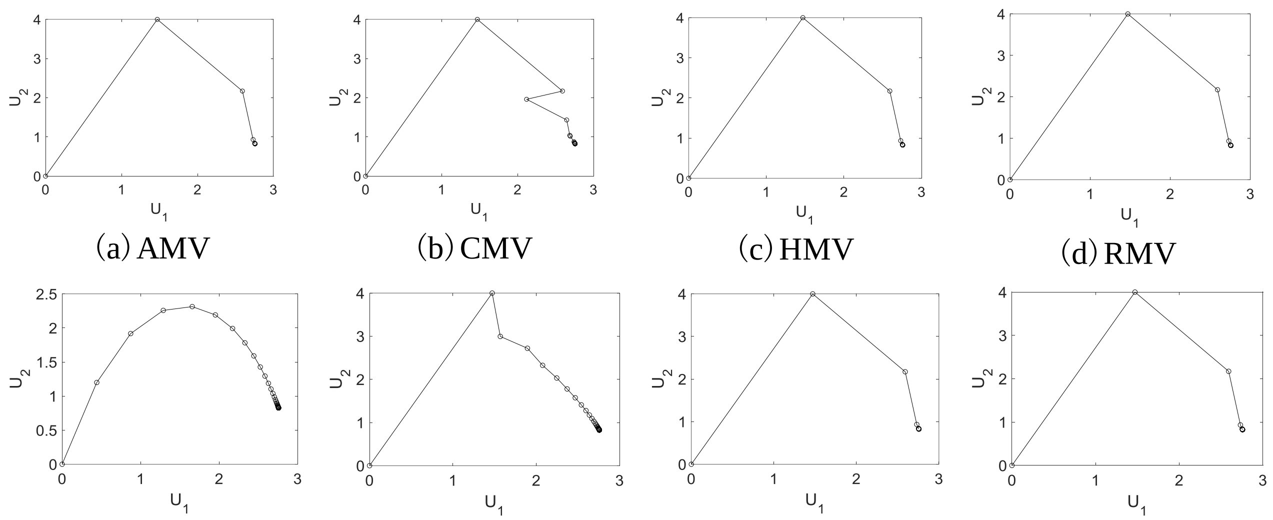
$$g(\boldsymbol{X}) = -\exp(X_1 - 7) - X_2 + 10 \tag{19}$$

When the simulation number of LHS method is 1×10^5, the obtained reliability indexes are respectively 2.8422, 2.8494, 2.8352, and the mean value is 2.8423.

Herein, we choose $\boldsymbol{C}=[1,0;0,1]$, the initial value of λ is 0.3. When the convergence condition is set to $\|\boldsymbol{X}_k - \boldsymbol{X}_{k-1}\| / \|\boldsymbol{X}_{k-1}\| < 1\times10^6$, the MPP point is (8.2058, 6.6605) and the reliability index is 2.8782. The number of iteration and the process of iteration are shown in Table 2 and Fig. 3 respectively.

Table 2 The number of iteration

Method	AMV	CMV	HMV	RMV	CC	MCC	HCC	ACC
Iteration	11 times	20 times	11 times	11 times	45 times	44 times	11 times	11 times



(a) AMV (b) CMV (c) HMV (d) RMV

(e) CC (f) MCC (g) HCC法 (h) ACC

Fig. 3 The process of iteration

Some conclusions can be drawn as Table 2 and Fig. 3:

(i) The iteration curve of AMV method, HMV method, RMV method, HCC method and ACC method are the same, and the iteration times are the same. The five kinds of algorithms have high efficiency and fast convergence.

(ii) The iteration curve of CMV method varies greatly, and the iteration times are larger.

(iii) The iteration curve of CC method is very smooth, but it has more iteration, while that of MCC method is less, but the iteration curve fluctuates greatly.

6. Highly nonlinear convex function

Assume the variables obey the normal distribution: $X_1 \sim N(4, 0.5^2)$, $X_2 \sim N(5, 0.4^2)$, the LSF is shown as follows

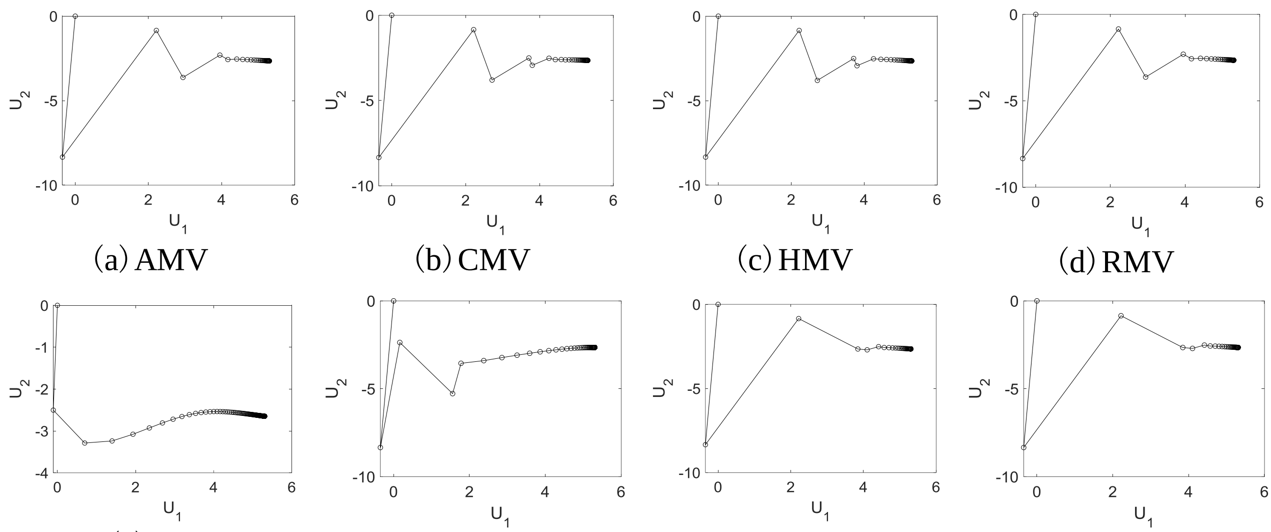
$$g(\boldsymbol{X}) = X_1^2 + 2X_2^2 - 5\exp(X_1 - X_2) \tag{20}$$

When the simulation number of LHS method is 5×10^6, the obtained reliable index is mostly infinite. Because the failure probability of this example is too small, and the simulation times of the LHS method are too large, this will reduce the calculation efficiency. Therefore, the LHS method is not suitable for the example with too small failure probability.

Herein, we choose $\boldsymbol{C}=[1,0;0,1]$, the initial value of λ is set to 0.3. When the convergence condition is set to $\|\boldsymbol{X}_k - \boldsymbol{X}_{k-1}\| / \|\boldsymbol{X}_{k-1}\| < 1\times10^6$, the MPP point is (6.6518, 3.9399), and the reliability index is 5.9289. The number of iteration and the process of iteration are shown in Table 3 and Fig. 4, respectively.

Table 3 The number of iteration

Method	AMV	CMV	HMV	RMV	CC	MCC	HCC	ACC
Iteration	67 times	69 times	68 times	67 times	200 times	77 times	66 times	66 times



(a) AMV (b) CMV (c) HMV (d) RMV (e) CC (f) MCC (g) HCC (h) ACC

Fig. 4 The process of iteration

Some conclusions can be drawn as Table 3 and Fig. 4:

(i) The HCC method and ACC methods have the least number of iterations, and the iteration curve is smooth, so the improvement effect is remarkable.

(ii) The AMV method and RMV method have a little more iteration, but the efficiency is still good.

(iii) The CMV method and HMV method have much iteration, and the iteration curve is not smooth.

(iv) The convergence curve of the CC method is very smooth, but the number of iterations is large. The number of iteration of the MCC method is reduced to less than 40% of the original number of iterations.

7. Highly nonlinear function

Assume the variables obey the normal distribution $X_1 \sim N(10, 5^2)$, $X_2 \sim N(9.9, 5^2)$, and the LSF is shown as follows

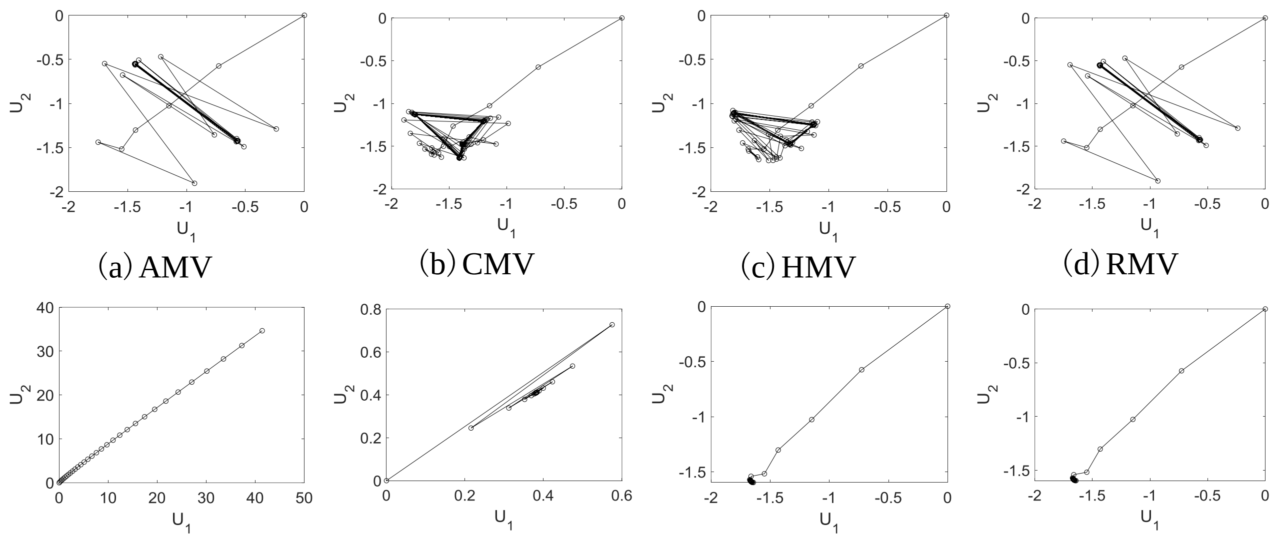
$$g(\boldsymbol{X}) = X_1^3 + X_1^2 X_2 + X_2^3 - 18 \tag{21}$$

When the simulation number of LHS method is 1×10^7, the obtained reliability indexes are 2.5247, 2.5214, 2. 5232, and its mean value is 2.5231.

Herein, we choose $\boldsymbol{C}=[0,-1;-1,0]$, the initial value of λ is 0.3. When the convergence condition is set to $\|\boldsymbol{X}_k-\boldsymbol{X}_{k-1}\|/\|\boldsymbol{X}_{k-1}\|<1\times10^{6}$, the MPP point is (1.6855, 1.9679), the reliability index is 2.2983. The number of iteration and the iteration curve are shown as Table 4 and Fig. 5, respectively.

Table 4 The number of iteration

Method	AMV	CMV	HMV	RMV	CC	MCC	HCC	ACC
Iteration	Oscillation	Oscillation	Oscillation	Oscillation	Non convergence	Oscillation	84 times	84 times



(a) AMV (b) CMV (c) HMV (d) RMV

(e) CC (f) MCC (g) HCC (h) ACC

Fig. 5 The process of iteration

Some conclusion can be drawn from Table 4 and Fig. 5:

(i) The stability of the three methods, AMV method, CMV method and HMV method, is gradually increasing, while the stability of RMV method is poor. However, none of the four algorithms converge.

(ii) The CC method and MCC method have poor convergence, but HCC method and ACC method have better convergence.

Different uncertain degree is chosen to analyze the influence of reliability index. The corresponding result is shown in Table 5.

Table 5 The reliability analysis with different uncertain degree

Stand deviation		Reliability index	
Uncertain degree	Range	Range	Uncertain degree
10%	(4.5,5.5)	(2.5536,2.0893)	10%
20%	(4.0,6.0)	(2.8728,1.9152)	20%
30%	(3.5,6.5)	(3.2832,1.7679)	30%
40%	(3.0,7.0)	(3.8304,1.6416)	40%
50%	(2.5,7.5)	(4.5965,1.5322)	50%

From Table 5, we can know that the greater the uncertainty of standard deviation, the greater the uncertainty of reliability index.

If the correlation is taken into account, with the correlation coefficient matrix $\boldsymbol{\rho}$, the update formula of sensitivity and reliability index is shown as follows

$$\alpha^{(k+1)}=-\frac{\boldsymbol{\rho}\nabla g(\boldsymbol{U}^{(k)})}{\sqrt{\nabla^{\mathrm{T}} g(\boldsymbol{U}^{(k)})\boldsymbol{\rho}\nabla g(\boldsymbol{U}^{(k)})}} \tag{22}$$

$$\beta^{(k+1)} = \frac{g(\boldsymbol{U}^{(k)}) - \nabla^{\mathrm{T}} g(\boldsymbol{U}^{(k)})\boldsymbol{U}^{(k)}}{\sqrt{\nabla^{\mathrm{T}} g(\boldsymbol{U}^{(k)})\boldsymbol{\rho}\nabla g(\boldsymbol{U}^{(k)})}} \tag{23}$$

In order to analyze the influence of correlation coefficient on the reliability index, different correlation coefficients were taken to calculate the reliability index. The corresponding result are shown in Table 6 and Fig. 6.

Table 6 The analysis of correlation coefficient

$\rho_{1,2}$	0.0	0.2	0.4	0.6	0.8	1.0
β	2.2983	2.0501	1.7829	1.6269	1.4247	1.2315

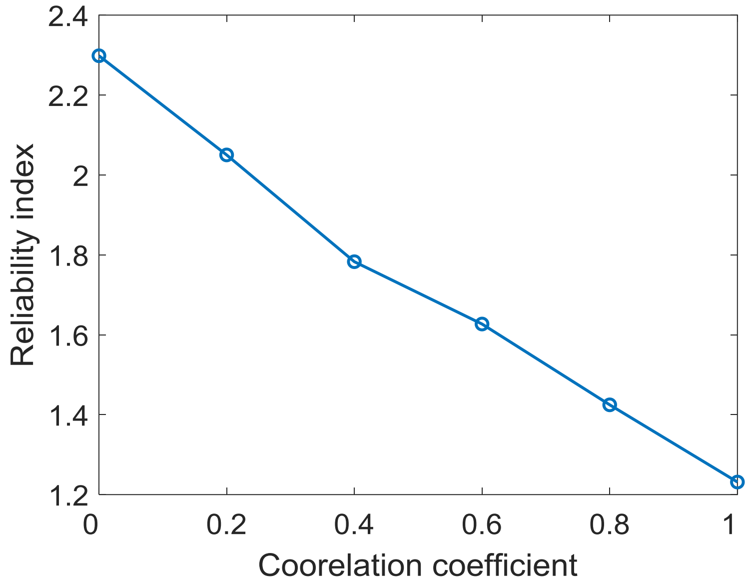


Fig. 6 The analysis of correlation coefficient

As can be seen from Table 6 and Fig. 6, when the two variables are positively correlated, the larger the correlation coefficient is, the smaller the reliable index is.

8. Conclusion

This paper adopted AMV method, CMV method, HMV method, RMV method, CC method, MCC method, HCC method and ACC method to analyze the reliability and compare the iterative curves with each other. Furthermore, we can draw some conclusion:

(i) When the AMV method is used to solve the convex function examples, the number of iteration is less. However, when CMV method is used to solve examples of the concave functions, the number of iteration is less. In general, the AMV method has less iteration than the CMV method.

(ii) The iteration times and iteration curve of AMV method and RMV are almost the same.

(iii) The convergence of AMV method, CMV method, HMV method and RMV method is poor, so it cannot solve the strong nonlinear examples.

(iv) The CC method has a smooth convergence curve, but it needs much iteration. Although the MCC method improves the convergence speed of the algorithm to a certain extent, the improvement effect of the HCC method and the ACC method are better.

(v) When the reliability index is large, the LHS method is difficult to calculate the reliability index, so the scope of its application is narrow and its practicality is poor.

(vi) The greater the uncertainty of standard deviation is, the greater the uncertainty of reliable index.

(vii) When the relationship between the variables is positive correlation, the larger the correlation coefficient, the smaller the reliable index.

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