Time-varying reliability analysis based on first order reliability method

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Abstract

The number of loading times and service life havs a great impact on the reliability index of the structure, so it is necessary to analyze the time-varying reliability of the structure. Because the HL-RF method and the Advanced Mean Value (AMV) method is easy and efficient to implement, they are adopted to analyze the time-varying reliability of the structures. The first two examples are used to analyze the time-varying reliability of loading times and service life respectively. The third example is adopted to fit the relationship between stress level and loading times and calculate the reliability index through the algorithm in this paper.

Keywords

HL-RF method; Advanced Mean Value Method; Regression Analysis; Time-variant Reliability.

1. Introduction

Time-variant reliability analysis is primarily used to evaluate the failure risk of structures throughout its lifetime, so as to avoid the casualties caused by the structural failure. Lots of experts developed some advanced methods which are applied to engineering problems [1-4]. Also, because the HL-RF method and Advanced Mean Value (AMV) method [5] is simple and efficient, it is usually nested inside the first-order reliability method (FORM). Hu et al. [6] improved the FORM method through EOLE and OSE extension methods, and applied the proposed FOSA method to the time-varying reliability analysis. The calculated results were very close to MCS method. Jiang et al [7] adopted the convex model and ellipsoidal model to analyze the time-variant reliability of dynamic reliability problems, and the correlation is taken into account. Hawchar et al [8] adopted the principal component analysis (PCA) and sparse polynomial chaos expansion (SPCE) to analyze the time-variant reliability. Wang et al [9] adopted the ellipsoidal model and Monte Carlo Simulation (MCS) method to analyze the time-variant reliability. This paper adopts the HL-RF method and the Advance Mean Value (AMV) method to respectively analyze the time-variant reliability about the time-varying reliability problems considering about load times and service life.

2. The iterative formula of HL-RF method

The transformation of random variables from X-space to U-space is shown as follows [10]

$$\boldsymbol{U}=\frac{\boldsymbol{X}-\boldsymbol{\mu}_X}{\boldsymbol{\sigma}_X} \tag{1}$$

where $\boldsymbol{\mu}_X$ and $\boldsymbol{\sigma}_X$ is mean value and standard deviation of normal variable $\boldsymbol{X}$, respectively; $\boldsymbol{U}$ is the variable that follows the standard normal distribution.

The updated formula for the sensitivity of the limit state function (LSF) is given as [11]

$$\boldsymbol{\alpha}^{(k+1)}=-\frac{\nabla^{\mathrm{T}}g(\boldsymbol{U}^{(k)})}{\left\|\nabla^{\mathrm{T}}g(\boldsymbol{U}^{(k)})\right\|} \tag{2}$$

The updating formula of the reliable index is shown as follows [11]

$$\beta^{(k+1)}=\frac{g(\boldsymbol{U}^{(k)})-\nabla^{\mathrm{T}}g(\boldsymbol{U}^{(k)})\boldsymbol{U}^{(k)}}{\left\|\nabla^{\mathrm{T}}g(\boldsymbol{U}^{(k)})\right\|} \tag{3}$$

The update formula of AMV method is shown as follows

$$\begin{cases} \boldsymbol{U}^{(k)}=\boldsymbol{0} & k=1 \\ \boldsymbol{U}^{(k+1)}=\beta^{(k+1)}\boldsymbol{\alpha}^{(k+1)} & k>1 \end{cases} \tag{4}$$

3. Loading Times

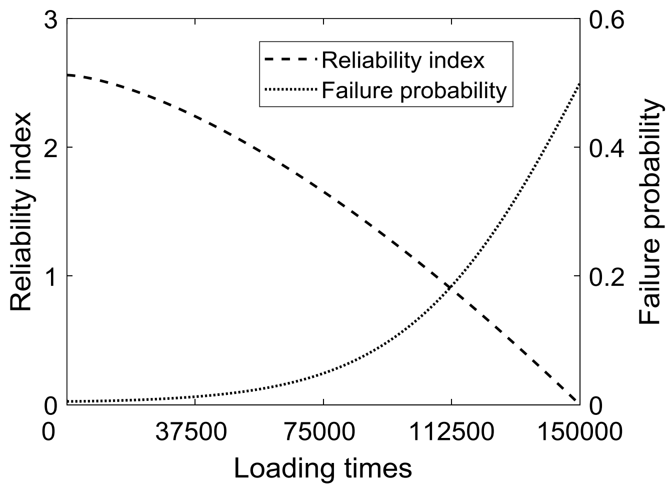
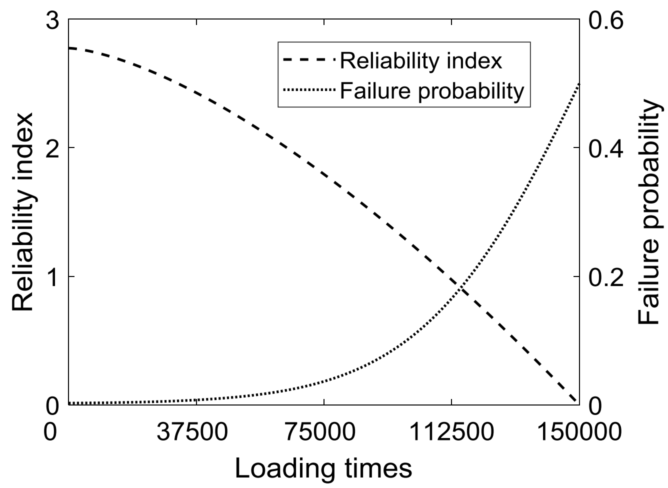
When the coefficient of variation (c.o.v.) of load is small, it can be known from the residual strength model that the residual strength after loading w times is [12, 13]

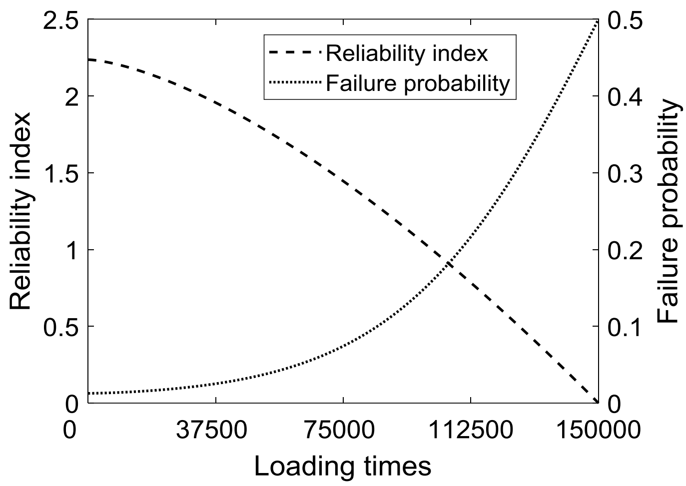
$$\sigma(w)=\sigma_0-(\sigma_0-S)\left(\frac{w}{N_S}\right)^{C_0} \tag{5}$$

where the initial strength is σ_0 MPa; the stress is S MPa; N_S is the fatigue life at the stress level of S, and the material constant is C_0.

Assume the material constant C_0 =1.5, the fatigue life N_S =150 000. Besides, the reliability index and the failure probability are respectively calculated with different standard deviation. The corresponding curve of the reliability index and the failure probability is shown in Fig. 1.

Table 1 The curve of reliability index and failure probability

Parameter	Curves
$\sigma_0 \sim N(600,60^2)$ $S \sim N(400,50^2)$ C_0 =1.5 N_S =150 000	
$\sigma_0 \sim N(600,60^2)$ $S \sim N(400,40^2)$ C_0 =1.5 N_S =150 000	

<table>
<tr>
<td>$\sigma_0 \sim N\left(600, 80^2\right)$
$S \sim N\left(400, 40^2\right)$
$C_0 = 1.5$
$N_S = 150\ 000$</td>
<td>
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Some conclusion can be draw from Table 1:

(i) Whether the standard deviation of the strength or stress is increased, the reliability of the structure will be reduced.

(ii) With the increase of loading times, the reliability index gradually decreases, while the failure probability gradually increases.

(iii) When the loading times reach the fatigue life, the reliability index is 0 and the failure probability is 50%.

4. Service life

Assuming the service life of the structure is T years, the strength of the structure in the nth year is

$$\sigma(n) = \alpha(n,k)\sigma_0 \tag{6}$$

In the power function model (PFM) and the exponential function model (EFM), the attenuation function is given as [14]

$$\alpha(n,k_1) = \left(1 - k_1 \frac{n}{T}\right)^r,\ k_1 = 1 - \alpha^{1/r}(T) \tag{7}$$

$$\alpha(n,k_2) = \exp\left(-k_2 \frac{n}{T}\right),\ k_2 = -\ln\left[\alpha(T)\right] \tag{8}$$

In the power function model, when r = 1, it is a linear function model (LFM); when r = 2, it is a quadratic function model (QFM); when r = 3, it is a cubic function model (CFM).

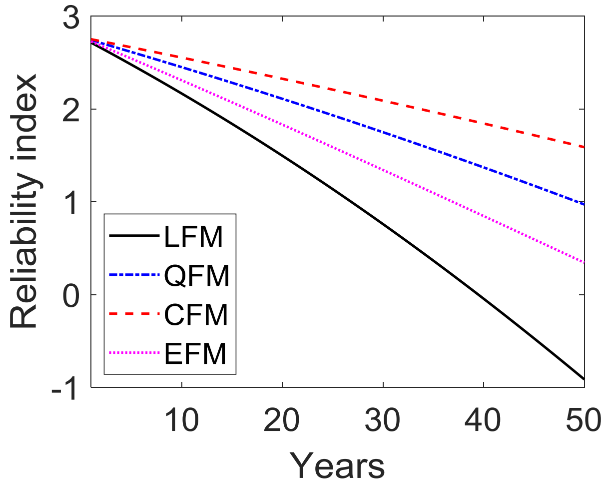
When the service life is 50 years, the parameter of $\alpha(T)$ can be set to 0.7, 0.8, 0.9. Assuming $\sigma_0 \sim N\left(600, 60^2\right)$, $S \sim N\left(400, 40^2\right)$, when $\alpha(T)$ takes different coefficients, the corresponding curve of reliability is shown as Fig. 1.

We can draw some conclusion from Fig. 1:

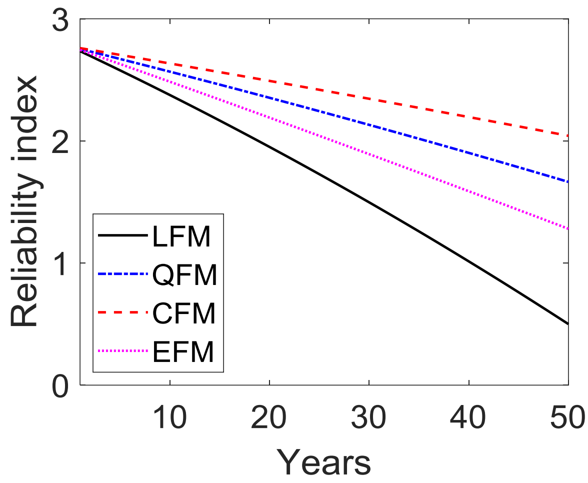
(i) The reliability index increases sequentially according to LFM model, EFM model, QFM model and CFM model.

(ii) In the four models, the reliability index decreases when the number of years increases.

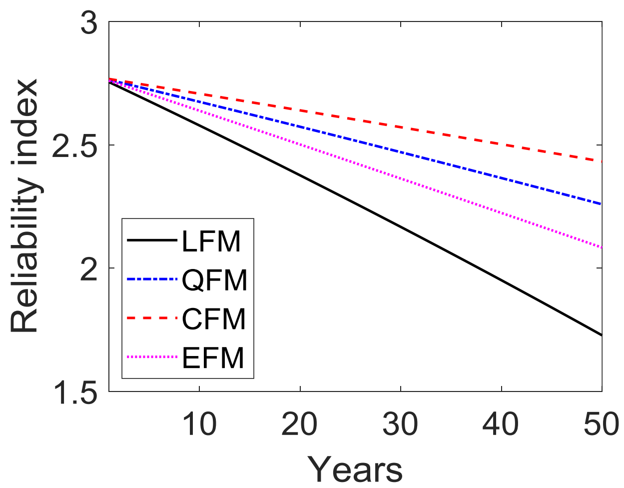
(iii) The larger the attenuation coefficient is, the smaller the variation of the reliable index.



（a） $\alpha(T)$=0.7



（b） $\alpha(T)$=0.8



（c） $\alpha(T)$=0.9

Fig. 1 The influence of $\alpha(T)$ on reliability index

5. Fatigue life Experiment

In order to find the relationship between the fatigue life distribution and the stress level of hot-rolled steel Q345 (16Mn), the stress levels are set at 394, 373, and 344 MPa, respectively, and 15 fatigue tests are performed. The experiment data is shown as Table 2.

Table 2 The data of fatigue life experiment under constant amplitude cyclic load

Stress level S (MPa)	Sample number	The mean of life μ (Number of loading times)	The stand deviation of life σ (Number of loading times)
394	15	113893	15130
373	15	196720	27322
344	15	722200	125800

According to the data in Table 2, Fig. 2 can be drawn in logarithmic coordinate system.

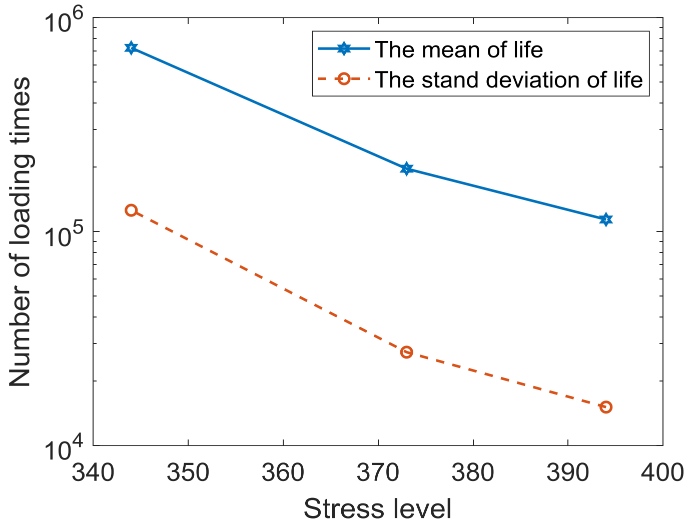


Fig. 2 The raw data in logarithmic coordinate system

The linear regression analysis is achieved by the least square method. Besides, the coefficient of determination is generally between 0 and 1, and the closer it is to 1. The more reliable, the regression equation is.

The coefficient of determination is 0.9807, so the regression equation is very reliable. Moreover, the regression equation is shown in Eq. (9). The contrast between the raw data and the regression data is shown in Fig. 3.

$$\ln \mu = 26.3009 - 0.0374S \tag{9}$$

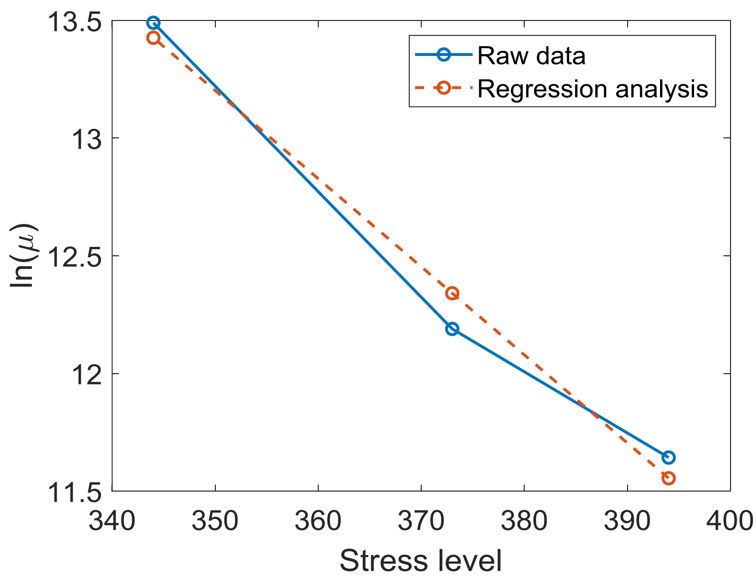


Fig. 3 The contrast between the raw data and the regression data

The coefficient of determination is 0.9753, so the regression equation is very reliable. Moreover, the regression equation is shown in Eq. (10), and the contrast between the raw data and the regression data is shown in Fig. 4.

$$\ln \sigma = 26.4487 - 0.0430S \tag{10}$$

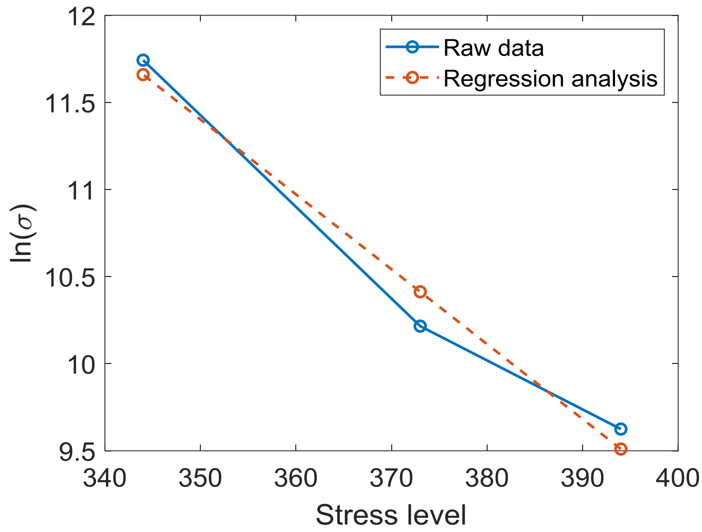


Fig. 4 The contrast between the raw data and the regression data

Assume the stress level S is 170MPa, and the life expectancy is $n \sim N(150000, 100^2)$. The reliability index of the structure is 2.2342, and the failure probability is 0.0127.

6. Conclusion

This paper adopts the HL-RF algorithm and AMV method to analyze the time-variant reliability. Besides, according to the analysis in this paper, the following conclusion can be drawn:

(i) Whether the standard deviation of strength or stress increases, the reliability of the structure will be reduced.

(ii) The reliability index increases sequentially according to LFM model, EFM model, QFM model and CFM model.

(iii) In the four models, the reliability index decreases when the number of years increases. Moreover, the larger the attenuation coefficient is, the smaller the variation of the reliable index.

(iv) In the logarithmic coordinate system, the stress level is linearly related to the life.

Acknowledgements

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