The Study on New Facility Location for Minimizing the Maximum Load

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Abstract

In this paper, we consider the new facility location problem in a general network with positive edge lengths and determined vertex weights. There are some facilities in the network already, we intend to minimize the maximum load in the facilities by adding a new facility. Based on the proximity principle and the rule of critical points, the continuous infinite points on the network graph can be discretized into finite candidate points, denoted as P, and a polynomial algorithm with a time complexity of O(mn2) is designed. Finally, a numerical example is given.

Keywords

New facility location; Maximum load; Network graph; Algorithm design.

1. Introduction

With the continuous growth of population, public facilities or chain facilities in cities are often overcrowded. For example, in large hospitals, people spend a long time in medical treatment and doctors are exhausted due to overwork at the same time, so the layout and planning of the original service facilities can no longer meet social needs. Because the original facilities are affected by geography and other factors, it is difficult to solve the contradiction by modifying the existing facilities, so the load of the facilities can only be reduced by adding similar facilities. How to minimize the maximum load of all similar facilities by planning the location of new facilities in the existing environment becomes an urgent problem to be solved.

The classical location problem includes three major theories: center [1], median [2] and cover [3]. On this basis, through the continuous expansion of scholars, derived a variety of variation problems, such as the center-median [4] problem, the gradual coverage problem [5] and so on. An implicit assumption in the above literature is that no other facilities exist in the environment or the influence of existing facilities on the location of new facilities is ignored at the time of location. When same type facilities already exist in the environment, the method by adding new facilities is called new facility location. In the existing studies, there are two types of new facility location according to location objectives, one is the model of competitive facility location, the other is the conditional location problem. The competitive facility location is to maximize the market share of its own chain facilities by adding new facilities when there are competitors in the market. Drezner [6] used computer simulation to study the competitive facility location of a single new facility in the plane. Fernandez et al. [7] compared and analyzed the advantages and disadvantages of the heuristic algorithm and the global optimization technology for single competitive facility location in the plane. Blanquero et al. [8] studied the model of a single competitive facility location in the network graph under gravity rules, and proposed a branch and bound algorithm based on interval analysis and *DC* optimization. Grohmann et al. [9] used meta-heuristic algorithm and nonlinear optimization method to solve the location problem of multi-competing facilities in the network. Different from the objective of competitive facility location, the conditional location problem aims to achieve the same optimization objective as the classical location problem (such as minimizing the maximum weighted distance, minimizing the sum of distance, etc.) by finding location for new facilities. The term "condition" refers to conditions in which similar facilities already exist in the environment. In the existing literature, Lin [10] and Handler et al. [11] first studied the conditional location problem. Minieka [12] expanded the unconditional single facility location problem and studied various conditional 1-center problems and conditional 1-median problems in the network. Berman and simchi-levi [13] solved the conditional p-median and p-center problem in the network graph by solving the unconditional (p+1)-median or (p+1)-center problem. Compared with the methods of Berman and simchi-levi [13], Berman and Drezner [14] proposed a simpler solution algorithm only by modifying the distance matrix. For continuous and discrete conditional p-center problem, Chen et al. [15] proposed a new relaxation algorithm. Kaveh and Nasr [16] used the improved harmony search algorithm (meta-heuristic algorithm) to apply the conditional p-center location to the actual location problem.

In the existing literature, the new facility location is either aimed at optimizing the distance or aimed at maximizing the profit of facilities. In this paper, the load level of the maximum load facility in the facilities is considered. By adding a new facility in the network, the load of the maximum load facility in all the facilities is minimized.

2. Models and Definitions

Urban traffic map can be abstracted as an undirected connected simple network graph G, $G=(V,E)$, where $V=\{v_1,v_2,...v_n\}$ is the set of vertices and $E=\{e_1,e_2,...,e_m\}$ is the set of edges, representing population gathering point (demand point) and highway respectively. We use the notation w_i to denote the weight of each vertex $v_i \in V$, $|V|=n$. Each edge $e \in E$ is associated with a positive number $l(e)$ (called the length of e), $|E|=m$. We also use the notation G to denote the set of all points on G. The distance between any two points x and y on G, which is represented by $d(x,y)$ (note that $d(x,y) = d(y,x)$), is the length of the shortest path on G from x to y. In this paper, as in most previous studies, we assume that the distance between any two vertices is known.

In the network G, there are already r same type old facilities $F=\{f_1,f_2,...,f_r\}$, providing the similar service or selling the same commodity. $X=\{x_1,x_2,...,x_r\}$ corresponds to the location of facility F in G, and each demand point follows the "proximity principle" to go to the corresponding facilities to receive service. The distance from point v_i to facility F is expressed as

$$d(v_i,F)=\min_{1\le k\le r} d(v_i, f_k) \tag{1}$$

$d(v_i,f_k)$ represents the shortest distance from demand point v_i to facility f_k. The nearest facility of point v_i is expressed as

$$f_i^* = \arg\min_{1\le k\le r} d(v_i, f_k) \tag{2}$$

Note that f_i^* represents the nearest facility or one of the nearest facilities from point v_i. If a demand point corresponds to multiple nearest facilities at the same time, then the weight of the demand point is evenly distributed to multiple facilities. Such demand points are called "evenly distributed demand points". The set of demand points served by facility f_k is expressed as

$$V_k = \left\{ v_i \in V \mid f_k = f_i^* \right\} \tag{3}$$

The load of this facility is

$$M_k = \sum_{V_k} \frac{1}{\alpha_i} w_i \tag{4}$$

Where $\alpha_i \in N_+$ is the nearest number of facilities at demand point v_i. In this paper, facility capacity constraints are not considered, and facility load only represents the degree of busyness and crowdedness of the facility.

The layout of the existing facilities in the network cannot be changed for a long period of time. The decision maker plans to change the load status of the existing facilities by adding a new facility f_o, with the target of minimizing the load of the maximum load facility. After the new facility is added, the facility set is expressed as $F_\#=\{f_1,f_2,...,f_r,f_o\}$. If facility f_o is built at a fixed location x, the load of the maximum load facility in facilities $F_\#$ is expressed as

$$L(x) = \max_{k=1,2\ldots r,o} M_k(x) \quad (5)$$

Where $x \in P$ is the set of candidate points of the newly added facility f_o on network G (how to obtain the point set P will be given in the next section). $M_k(x)$ represents the load of facility f_k when facility f_o is at location x, $f_k \in F_{\#}$. The minimum maximum load objective is as follow

$$L(x_{opt}) = \min_{x \in P} L(x) \quad (6)$$

$L(x_{opt})$ is the load of the maximum load facility in the facilities when the new facility f_o is in the optimal location x_{opt}, $x_{opt} \in P$. Our target is to find location x_{opt}, where $L(x)$ is the lowest.

3. Properties and Analysis

The edge of the network is composed of a series of continuous points, and the number of these points is infinite. How to discretize these continuous points into finite points with different properties according to the nature of the problem, and make these points represent all the points G is the content to be studied in this part. Because of each demand point in the network follows the "proximity principle", the distribution of demand points has nothing to do with the weight of the demand point itself, but with the distance from the demand point to the nearest facility. In this paper, the points with a distance of β_i from point v_i on edge of the network are called critical points, denoted by *NIP* [17]. *NIPs* represents the set of all *NIP* points in the network. β_i is the critical distance of demand point v_i, which is equal to the distance from the demand point to the nearest old facility. It is also the maximum distance that demand point v_i is willing to go to the new facility f_o. There is

$$\beta_i = d(v_i, F) \quad (7)$$

In fact, *NIP* point is the boundary point defined by demand point v_i as the "center" and critical distance β_i as the "radius". Because of the existence of network branches, a demand point corresponds to multiple NIP points on multiple edges, but a *NIP* point corresponds to only one specific demand point, unless the *NIP* points of multiple demand points coincide.

Lemma 1. The total number of *NIP* points on the network graph is at most $O(mn)$.

Proof. For a demand point $v_i \in V$, the demand point generally corresponds to only one *NIP* point on a edge. In the special case, there will be two *NIP* points, where the demand point pass through the vertex on both sides of the edge and the shortest distance to the two points on the edge is equal. In addition, there is also the case that the critical distance range β_i of point v_i cannot reach some edges on G, where the demand point has no corresponding *NIP* point on these edges. If a demand point has a corresponding *NIP* point on all the edges of the graph G, since there are m edges on the network, then a demand point has at most $O(m)$ *NIP* points on the network graph. Therefore, all demand points V have at most $O(mn)$ *NIP* points on the network graph. That is, the $|NIPs|$ are at most $O(mn)$. Thus,the lemma follows.

Let all *NIPs* plus all demand points V definited as *NIPS*, that is, NIPS=$NIP \cup V$. Then the whole network is divided into many small line segments by *NIPS* and facility points F. A class of line segments within the critical distance of one or more demand points is defined as *SEC*. The line segment that is not covered by the critical distance of any demand point is defined as an "invalid segment". Note that both the *SEC* segment and the "invalid segment" referred to in this paper do not contain two endpoints of the segment.

Lemma 2. The new facility cannot be located within "invalid segment".

Proof. Any point in the "invalid segment" exceeds the maximum acceptable distance β of all demand points on the network graph. If the new facility f_o is located within such a segment, under the "proximity principle", no demand point in the network will go to facility f_o to receive services, and then the location is meaningless. Thus,the lemma follows.

According to lemma 2, in addition to the "invalid segment" on the edge, it is also necessary to exclude the location X of the existing old facility, because the new facility cannot coincide with the location of the existing facility. Let all "invalid segments" in the network defined as the set U, then $G'=G\backslash(U \cup X)$ is the feasible location range for the new facility. In addition, in order to explain the characteristics of some points on a line segment and facilitate the use of the following, the concept of "equivalent points" is first introduced, that is, when a new facility moves on a line segment, the facilities to which each demand point on the network graph goes remain unchanged, and all points on this line segment are called "equivalent points".

Lemma 3. All points in a single line segment are "equivalent points".

Proof. When the new facility f_o is built at any point within the *SEC* segment, the demand points corresponding to the critical distance range will go to facility f_o to receive services, and the new facility f_o will "cover" the same demand points. At the same time, the demand points served by the facility in $\{F_{\#}\backslash f_o\}$ also remain unchanged, that is, if the new facility f_o is at any point within the *SEC* segment, it will not change the distribution of all demand points V on the network graph. Therefore, the points in the line segment are equivalent points for new facility location. Thus,the lemma follows.

Different from lemma 3, when the new facility f is built at the end point of the *SEC* line segment, if the end point is *NIP* point, the distance from the demand point corresponding to *NIP* to the new facility f_o is the same as that from the demand point to the previous old facility. This paper adopts the rule of equipartition critical point [6,18]. That is, when the new facility is located at the *NIP* point of a demand point, the demand point weight is evenly distributed among all the nearest facilities. Based on the critical point rule, the *SEC* in the network can be divided into two categories. The two ends of the *SEC* segment of the first kind are different *NIP* points, or one end is *NIP* point and the other end is facility point f. The properties of the inner point and the end point of a line segment are different. In the second kind of *SEC*, at least one endpoint is demand point v, and the inner point of such a line segment has the same property as demand point v. Here, the "property" is reflected in the different load of the new facility at different locations.

Through the above analysis, the entire network can be composed of *NIPS* point, facility point F, *SEC* segment and "invalid segment". Taking the midpoint of the first kind of *SEC* line segment as the representative point of the *SEC* segment, called *MP* point, and represent all "equivalent points" in the *SEC* segment. Let all *MP* points on the network graph are denoted as *MPs*. For the second kind of *SEC*, the demand point v at one end represents the point v itself and the inner point of the *SEC*. From the perspective of facility location effect and function, point set ($NIPS\backslash X$) and point set *MPs* are candidates for new facility. So here's the definition $P=(NIPS\backslash X)\cup MPs$. The original location range G' is "equivalent" to point set P, $P\in G'$. Then the continuous points on the edge of the network are discretized into finite candidate points with different properties.

We have obtained the set of candidate points, so the steps of the *NFL*(new facility location) algorithm can be summarized as follows.

Step 1. Determine the point set *NIPs*, and form the point set *NIPS* with the demand points V.

Step 2. Determine the point set *MPs*, and then determine the candidate points P.

Step 3. Calculate the load of each facility when the new facility f_o is built at point $x\in P$ and determine the maximum load facility.

Step 4. Repeat Step3 to determine the maximum load facility of each candidate point in P.

Step 5. By comparation, the candidate point corresponding to the maximum load facility with minimum load is the optimal location x_{opt}.

Theorem 1. If the weight of each demand point is a given value, the time complexity of the *NFL* algorithm is $O(mn^2)$.

Proof. According to the previous discussion, Step 1 takes $O(mn)$ time, Step 2 takes $O(mn)$ time. Using equations (1) to (5), Step 3 takes $O(n)$ time. The algorithm complexity of Step4 is $O(mn^2)$, the algorithm complexity of Step4 is $O(mn)$. Therefore, the time complexity of the whole algorithm is $O(mn^2)$.

Theorem 1 shows that the problem of minimizing the maximum load can be solved in time $O(mn^2)$.

4. Numerical Example

The network graph G is shown in figure 1. There are already two facilities f_1 and f_2 in the network, which need to serve 12 demand points. The weight of each demand point is w_1=6, w_2=5, w_3=8, w_4=7, w_5=10, w_6=6, w_7=7, w_8=5, w_9=9, w_{10}=9, w_{11}=11, w_{12}=5. The length of each edge is given in the network. The decision maker intends to build a new facility f_o to alleviate the overloading of existing facilities.

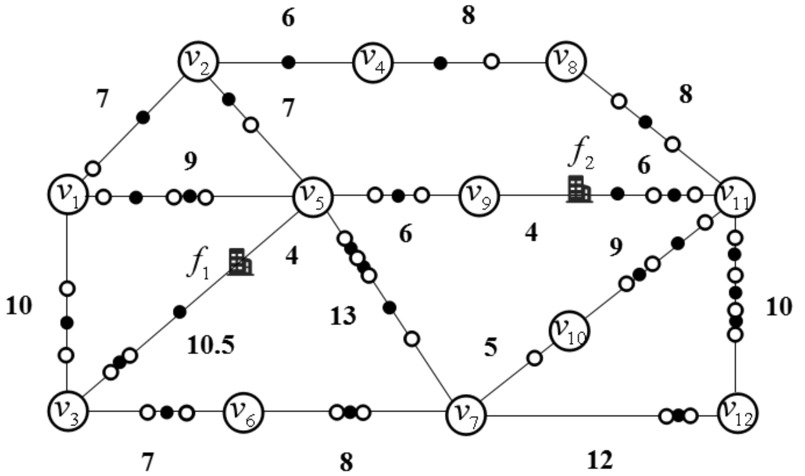

Figure 1. An example of network G

According to the *NFL* algorithm, the location steps for a new facility are as follows.

Step 1. All *NIP* points are marked with hollow dot in the graph G. The demand point set V plus the *NIPs* point set forms the *NIPS* point set.

Step 2. All *MP* points are marked with black dot in the graph G. Therefore, the candidate point of the new facility is the sum of all the hollow dot, the black dot and the demand point in the network.

Step 3. According to "proximity principle", if the new facility is built at point v_1, then the load of each facility is M_o=32, M_1=17, M_2=39. The maximum load facility is f_2 .

Step 4. By repeating step 3, we can obtain the load of the maximum load facility when new facility at different candidate points, as shown in the table 1.

Step 5. By comparing the calculation results in table 1, the candidate point corresponding to the minimum "maximum load" value is v_6. So v_6 is the optimal location for the new facility f_o .

Table 1. The maximum regret values

Candidate point	Maximum load	Candidate point	Maximum load	Candidate point	Maximum load	Candidate point	Maximum load
v_1	39	NIP_9	39	v_4	34	MP_{19}	42
NIP_1	39	MP_7	39	MP_{14}	37	NIP_{26}	40.5
MP_1	39	NIP_{10}	34.5	NIP_{18}	39.5	NIP_{27}	39
v_2	36.5	v_6	30	v_8	42	MP_{20}	42
NIP_2	39	NIP_{11}	32	NIP_{19}	39.5	NIP_{28}	42
MP_2	39	MP_8	36	MP_{15}	44	v_{12}	42
NIP_3	39	NIP_{12}	36	NIP_{20}	31.5	NIP_{29}	42
v_3	39	v_7	36	v_9	49	MP_{21}	42

NIP_4	39	MP_9	39	MP_{16}	49	NIP_{30}	40.5
MP_3	39	NIP_{13}	39	NIP_{21}	45.5	NIP_{31}	40.5
NIP_5	39	NIP_{14}	39.5	MP_{17}	42	MP_{22}	42
MP_4	43	MP_{10}	44	NIP_{22}	40.5	NIP_{32}	45.5
NIP_6	39	NIP_{15}	47	v_{11}	44	MP_{23}	49
v_5	39	MP_{11}	50	NIP_{23}	36	NIP_{33}	45.5
NIP_7	39	NIP_{16}	36	v_{10}	36	MP_{24}	42
MP_5	39	MP_{12}	36	NIP_{24}	36	NIP_{34}	42
NIP_8	39	NIP_{17}	36	MP_{18}	43	\	\
MP_6	39	MP_{13}	34	NIP_{25}	40	\	\

5. Conclusion

In this study, we have considered the new facility location problem in a general network. The objective is to minimize the maximum load of all similar facilities by planning the location of a new facility. Our study provides a novel idea for the location of public facilities in cities, such as hospital, which has certain theoretical and practical significance. In the future study, the facility location problem in uncertain environment can be further considered.

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