Sandcastle Erosion Model (SEM) Based On Cellular Automaton

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Abstract

In this paper, I seek to establish a model that will quantificationally reveal how the shape and water content of sandcastle foundation influence its lifespan. Firstly, I combine Cellular Automaton with thought of differential and build the Sandcastle Erosion Model (SEM). Based on the aforementioned SEM, I set a liquid bridge between two adjacent sand grains which generate adsorption force (Fad), then introduce the parameter adhesive contact angle φ, thus connect water-sand mixing ratio with collapse time. I carry out emulation and conclude the best shape and water-sand mixing ratio to build the most stable sandcastle.

Keywords

The best sandcastle foundation, Cellular Automaton(CA)，Sandcastle Erosion Model(SEM), Soil mechanics.

1. Introduction

Creating sandcastle, which originated in the United States more than 100 years ago, is becoming a recreational activity that flourishes all over the world. Whenever you stroll along the coastline, what comes into sight must be plenty of tourists building elaborate sandcastles with their extraordinary imagination. However, different sandcastles have various stability. Therefore, we seek to establish a model that will reveal how does the shape and water content of sandcastle foundation influence its lifespan, so as to help everyone make a more stable sandcastle in a well-grouped way.

2. The Best Three-dimensional Geometry of a Sandcastle

2.1 Assumptions

◆The tides and waves are the same all the time leaving out the influence of season, weather and other factors.

◆The boundary of the water bridge between two adjacent sand grains is approximately an arc which is tangent to these two sand grains.

◆The water contains no impurities and the temperature is constant, so the surface tension coefficient of the water is the same everywhere.

2.2 Sandcastle Erosion Model (SEM)

As the three-dimensional model are much more complex than 2-bit models, I intend to use two dimensions model to simulate the condition of three dimensions to simplified calculation.

According to the idea of differentiation, I cut the three-dimensional space into enough thin slices along the vertical direction (the thickness is the side length of a cube cell), and each slice which approximates a two-dimensional plane can be divided into more enough cells. To predigest the force analysis and the types of state change, I take the cube cell here and the state of each cell is either water or sands. One cell space contains nine cells, and I choose the Moore neighbor in which each cell is surrounded by 8 neighbors.

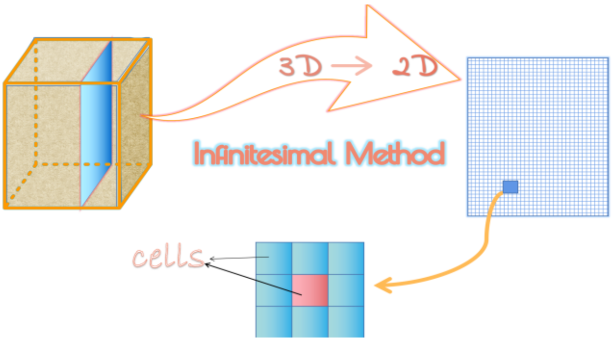

Figure1 Two-dimensional Cell Space

For using sand of the same type, same quantity, and sand-to-water mixture proportion in each sand foundation, I approximate that the every sand take the identical viscous force in the inside of each sand foundation, and neglect it. So I only discuss the difference that the angle between the surface of every foundation and the flow direction.

Therefore, for a certain sand cell, I only discuss its own gravity, the pressure of the upper cell on it and the force of water on it. The force of water on the sand cell can be decomposed into the impact force perpendicular to the force surface of the cell and the viscous force along the force surface of the cell. As shown in the following figure:

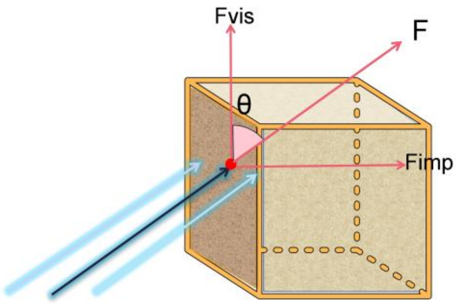

Figure2 Force Analysis of Sand Cell

Among them, the viscous force usually refers to the force that the object is subjected to when it moves in the viscous fluid. According to the relativity of motion, we can make inference that when the fluid is moving relative to the object, the object in the tangential direction is subjected to the same force as the viscous force, so we can also apply Stoker's law.It can be deduced from the momentum theorem of the fluid :

$$\begin{cases} Fim = \rho V_n^2 S_{cell} \\ Fivs = 3\pi\eta d V_t \end{cases}$$

※ ρ :Density of seawater

※ S_{cell} :The surface area of the cell side, that is the area of the whole force

※ η :Viscosity Coefficient of sea water

※ d : Cellular side length

※ V_t : tangential velocity of V

※ V_n :normal velocity of V

Since it has been set that each shape of the sand foundation bears the same water quality, that is, the same density, and the volume of each cell divided and each surface area are the same. So cells are both constant in the expression of $Fimp$, and the value of $Fimp$ is only associated with the normal velocity V_n.

According to the above mechanical expression:

$Fimp$ is proportional to the square of the normal velocity, the normal velocity is the component of the combined velocity in the direction of the vertical cell surface, that is,

$$V_n = V \times \sin\theta$$

※θ： the Angle of the sand carving relative to the horizontal plane, which can be approximately considered as the angle between the cellular stress plane and the horizontal plane.

According to the assumption that the direction of water flow is horizontal and uniform at a certain time, the V in the normal velocity V_n expression is the same when simulating the erosion of water flow for different shapes of sand foundation. Therefore, V_n is only related to the inclination of the sand foundation. In other words, the impact force $Fimp$ is only related to the inclination of the sand foundation. So we get that the influence of the shape of sand foundation on the erosion process of sand foundation: the properties of inclination by affecting $Fimp$ to influence the sand cellular state transition probability That is, it change the rules of cellular automaton keeping static or falling probability, and thus affect the dynamic erosion process Therefore, different shape of sand foundation have different lifespan.

From the above analysis, we define the cellular state transfer rule as follows:

For the sand cell in the center, when it is affected by the viscous force of water in the direction of its lead weight, compared with its own gravity, if the gravity is greater than the water viscous force $Fvis$, the sand grain has a downward trend (in the second question, the adsorption force of neighbor cell on it will be added).

After the sand cell takes impact of water, it may be stationary in the same place or fall. Its state transition probability is related to the number of sand cell neighbors in the water state and the angle between the water force and the stress surface of the sand cell, which reflects the influence of the sand foundation angle on the dynamic change process of erosion.

For a given grain of sand, there is a certain probability that it will disengage:

$$P_{fall} = \sin\theta \times \frac{n}{8}$$

$$P_{maintain} = 1 - P_{fall} = \sin\theta \times \frac{n}{8}$$

The transformation relations among various cellular spatial states are given, as shown in the following figure.

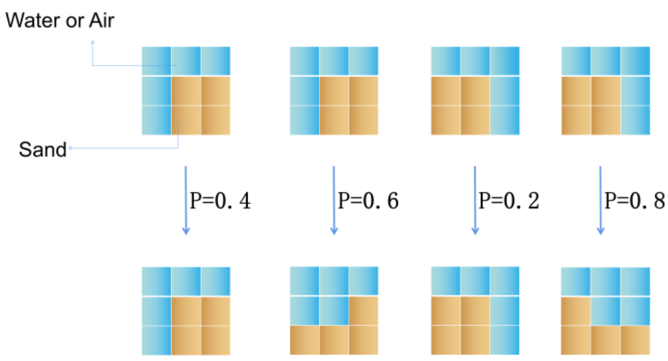

Figure 3 Transformation Rules

2.3 Conclusion

We respectively set the shape of the sand foundation as a cube, cylindrical, conical, and trapezoidal four prism with the same volume, and carried out the erosion simulation of water flow.

Table1 Shape and Hold Time

Shape	CUBE	CONE	Frustum of a prism
Hold time	8.33×10^4	1.88×10^5	1.21×10^5

By cellular automaton simulation, we come to a conclusion that conical sand foundation has the strongest ability to endure the impact of waves and tides, the ladder was the second best, and the cube was the worst.

Based on our result that the cone is the best geometry for the sand foundation, we also need to find the best angle of the cone.Under the condition that the volume, position and other other conditions of the conical sandcastle remain unchanged, we only change the inclination angle of the conical sand foundation. We obtain the relationship between the cone's inclination angle and the sand foundation's holding time by the cellular automaton model simulation,as shown in the following figure:

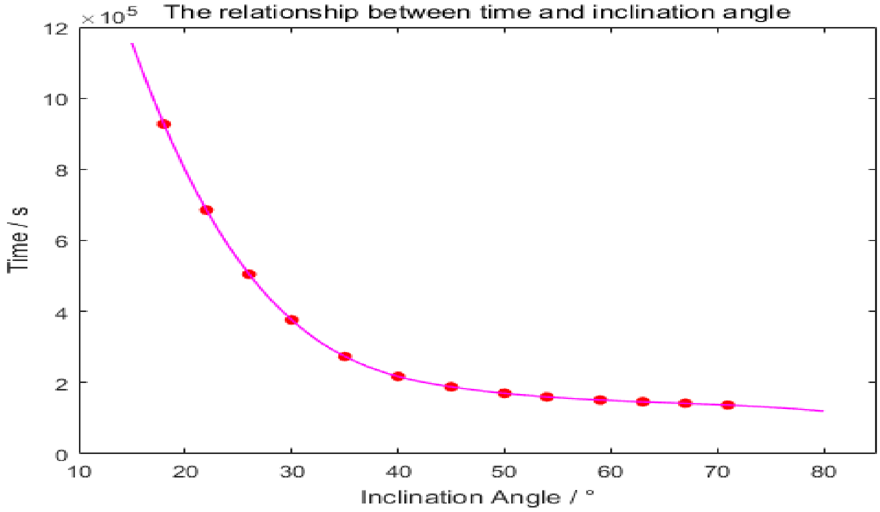

Figure4 The Relationship Between Time and Inclination Angle

As can be seen from the curve chart, when the dip angle is smaller, the sandcastle lasts longer, which is consistent with the previous analysis. Just like the beach we see, the beach's inclination will gradually decrease after a long period of wave and tidal erosion, and eventually close to 0 °. However, with the total volume of sand unchanged, we can't just continuously decline the angle of the sand foundation in order to extend the retention time, because the height of the sand foundation will also decrease with the decline of the angle. If the sand castle doesn't have a certain height, we can't create on it. According to the graph above, I find that when the dip angle of sand castle is greater than 40 °, the rate of decrease of sand foundation retention time slows down as the dip angle increases. Therefore, after considering the sand castle's height and retention time, I conclude that the conical sand foundation with an inclination of 60° is the most suitable geometric shape.

3. The Optimal Water-sand Mixing Ratio

Anyone who has built a sandcastle near the sea has probably had the experience: You cannot build a sandcastle with dry or too wet sand. Only with wet sand that contains a certain amount of water can you make it. After a sandcastle built, for keep it in its original shape, you must carefully water it to maintain a certain degree of humidity. However, when adding too much water, the sand particles will be separated by water and become a cloudy liquid. In these conditions, the slippage between the grains increases and the friction decreases, which will reduce the persistence of the sand foundation.

3.1 Assumptions

◆The sand is in an unsaturated state. Most soil in the nature is unsaturated with three states: the solid (soil particles), liquid (water), gas (air).

◆The boundary of the water bridge between two adjacent sand grains is approximately an arc which is tangent to these two sand grains.

◆The water contains no impurities and the temperature is constant, so the surface tension coefficient of the water is the same everywhere.

3.2 Liquid Bridge Model

Firstly, I set an arc-shaped liquid bridge with a boundary between two adjacent sand grains, and the radius of the circle is variable, so as to adjust the water content between the two Bridges, which is equivalent to adjusting the water-sand ratio. Then, I consider the relationship between with different water content and adhesive contact Angle from geometric calculation, that is, the greater the water content, the adhesive contact Angle formed by the liquid bridge. Then force analysis and the adsorption force between two contact sand (Adsorption Force) and adhesion contact Angle φ is the function of the independent variables, thus the moisture content and adsorption force between adjacent grains of sand, and get a moisture content - adsorption function expression. Next, the variable in the adsorption force expression defines a parameter called viscosity (Dad) to correct for the probability that the sand cells in the cellular automaton fall off after being impacted. The details are as follows:

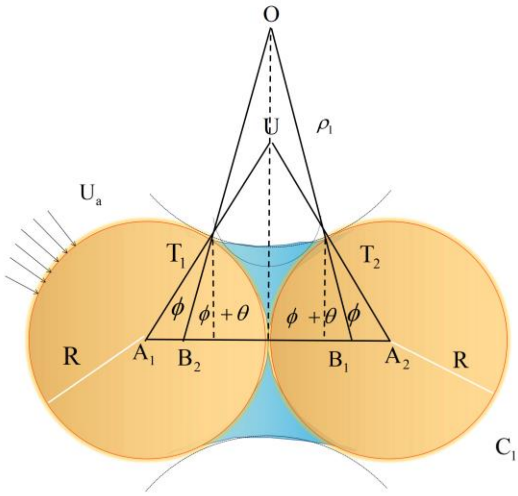

Figure5 Liquid Bridge

From the knowledge of fluid mechanics and geometry, we can figure out:

$$\rho_2^* = \left[\frac{1}{\rho_1^*} + \frac{\sin\phi_1 \sin(\phi_1+\theta) - \cos(\phi_1+\theta)(1+\cos\phi_1)}{\sin^2\phi_1}\right]^{-1}$$

ρ_1^* ρ_2^* are normalized principal radius of curvatures.

ϕ is adhesive contact angle. It refers to the Angle between the intersection point of water and sand grain and the line between the center of sand grain and the radius of two adjacent sand grains, which changes with the adjustment of water-sand ratio(Adjustment of water content).

θ is the contact angle between the liquid bridge and the particles, normally it is equal to 0°.

Then the surface tension is the adsorption between the two sand —Fad :

$$Fad = 2\pi\alpha R\sin\phi\left[\sin\phi + \frac{\sin\phi}{2} * \left(\frac{1}{\rho_1^*} - \frac{1}{\rho_2^*}\right)\right]$$

Fad is a function of φ and $\left(\frac{1}{\rho_1^*} - \frac{1}{\rho_2^*}\right)$. In the process of water content increasing from zero, the joint action of Angle φ and variable $\left(\frac{1}{\rho_1^*} - \frac{1}{\rho_2^*}\right)$ makes Fad increase first and then decrease.

We believe that the greater the adsorption force between sand grains, the less likely they are to fall off when they are subjected to water shock, so P_{fall} is negatively correlated with Fad.

$$P_{fall} = (k/Fad) \times \sin\phi \times (8/n)$$

※ k is the negative correlation coefficient between P_{fall} and Fad.

3.3 Simulation Results & Analysis

Through the simulation of cellular automaton model, the following results are obtained:

Table2 Water Content and Maintain Time

Water Content/%	2.14	4.29	6.43	8.57	10.71	12.86
Time/t	207410	233448	266142	310802	373460	464860
Water Content/%	15	21.41	27.84	34.26	40.69	47.12
Time/t	618648	464860	373460	310802	266142	233448

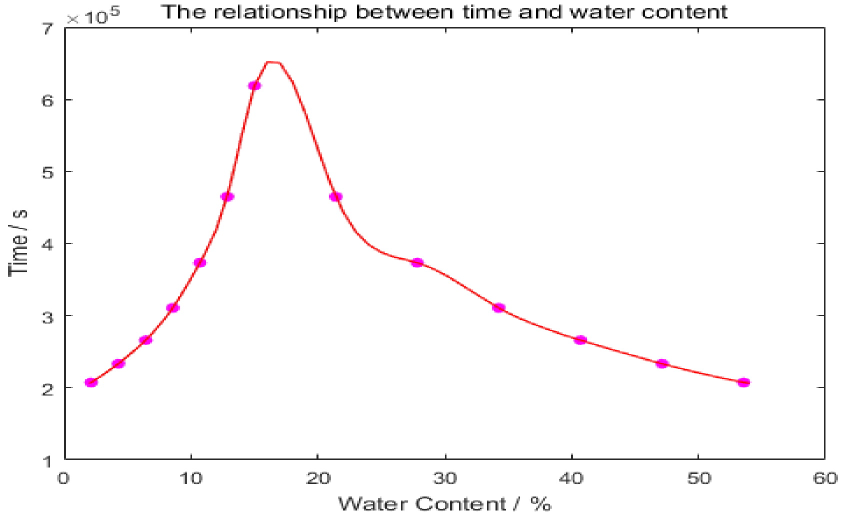

Figure6 The Relationship Between Time and Water Content

As we can see from the figure, as the water content gradually increases, the duration of the sandcastle's resistance to waves and tides will rise first, and then begin to fall when it reaches a certain point. After analyzing the results, we come to the conclusion that 15% is the best water content of sand castle, and the sand castle has the longest duration of resistance to seawater.

After consulting the literature, it can be seen that the sand with a certain water content will exhibit cohesion and its stability will increase. However, the lubrication effect of water with high water content reduces the friction force of sand, and thus reduces the stability of sand. It can be seen that the model results are in line with the actual situation.

4. Conclusion

I carry out emulation and conclude that cone with dip angle of 60°is the optimal shape to resist water erosion. I then carry out emulation and conclude that 15% is the best water content to resist water erosion. Finally, I discover that cone with dip angle of 45°~ 60° remains the optimal shape in case of rainfall scenario. Besides, I came up with other strategies (changing the type of sand, adding new material to the sand, etc.) to stabilize the sandcastle.

References

[1] https://en.wikipedia.org/wiki/Tide#Bathymetry

[2] https://en.wikipedia.org/wiki/Ocean_Wave

[3] https://en.wikipedia.org/wiki/Cellular_automaton

[4] https://zhidao.baidu.com/question/58841912.html

[5] Mu Qingsong, Wang Rusheng. The effect of liquid bridge gravitation on the shear strength of unsaturated soil[J]. journal of lanzhou university (Natural science edition)，2019,55(05):604-609.

[6] Zhou Xiaofeng. Matlab simulation of sand heap model of cellular automata[J]. a think-tank era,2019(41):195-196.

[7] Yang Chong,Zuo Shuangying. Study on the influence of continuous rainfall on the stability of soil slope [J]. roadbed construction, 2019(05):6-11.