Comparative Study of Static Electric Field, Steady Electric Field and Steady Magnetic Field

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Abstract

Start with the definition to discuss the law of the unity of opposites in each field, and then discuss the field lines, the field source of the excitation field, the constitutive equation of the medium, theorems and laws, and the similarities and differences of boundary conditions in different fields. Finally, the electric field and magnetic field are compared, analyzed and summarized in the form of a table. Through comparative analysis, find out the inner connection between knowledge, establish a knowledge network.

Keywords

Electrostatic field, Steady electric field, Steady magnetic field, Comparative study.

1. Introduction

All changes in nature are related to electricity and magnetism. Electromagnetism has penetrated into all fields of modern science and technology and has become the theoretical basis of many disciplines and technologies. As an important part of electromagnetism, electric and magnetic fields are both similar and different in nature. For each knowledge point, it is usually learned one by one in the traditional order, without too much explanation of the law between knowledge and knowledge, so that the understanding of knowledge is relatively isolated, and it is impossible to have a comprehensive understanding of electromagnetic fields, and it takes a lot of time and the effect is not ideal .

This paper adopts the comparative learning method to analyze the electrostatic field, steady electric field, and steady magnetic field together, finds out the law of the unity of opposites, and refines complex knowledge content into a knowledge network diagram that can clearly reflect the electromagnetic field theory system.

2. Comparative Study of Static Electric Field, Steady Electric Field and Steady Magnetic Field

2.1 Comparative Study on Definition

Both the static electric field and the steady electric field are electric fields excited by two states of charge, and the charge distribution of the steady electric field does not change with time. Therefore, the field strength of the steady electric field and the field strength of the electrostatic field are not time-dependent function[1]. At the same time, both the steady electric field and the electrostatic field are electric fields excited by electric charge according to Coulomb's law. This is the similarity between the two. Based on the similarities between the two, electrostatic field is often used as a special case of steady electric field to analyze and study.

2.2 Comparative Study on the Field Line

In order to vividly describe the distribution of the field strength, some artificial lines are artificially made, and these lines are used to reflect the characteristics of the field and describe the size and direction of the field strength at each point in the field. The field lines in the electric field are called electric field lines, and the field lines in the magnetic field are called magnetic induction lines.

The difference between electric field lines and magnetic induction lines is that electric fields are excited by electric charges, while magnetic fields are excited by magnetic poles. Since there are separate positive and negative charges in nature, the electric field lines start from positive charges and end at negative charges, and their lines of force are the non-closed curves. on the contrary, since the north and south magnetic poles in nature exist at the same time, their magnetic lines of force are the close curve. Because outside the magnet, the magnetic field lines start from the N pole and point to the S pole. Inside the magnet, the magnetic field lines point from the S pole to the N pole, forming a closed curve[2].

The common points of electric field lines and magnetic induction lines are[1]: (1) the tangent direction of each point on the line is consistent with the field strength direction of that point, and the direction of the line reflects the distribution of the field strength direction, (2) the density of lines The degree reflects the size of the field strength, (3) Any two lines do not intersect.

2.3 Comparative Study of Field Sources under Excitation

In a conductor in electrostatic balance, the internal additional electric field and the external electric field completely cancel out, so that the internal field strength of the conductor is zero everywhere, reaching a state of electrostatic equilibrium. The steady electric field is always accompanied by a steady current distribution in the conductor. It only requires that the charge distribution in the conductor does not change with time and achieves dynamic equilibrium[3]. Therefore, the field strength inside the conductor in the steady electric field is not necessarily zero, and the field strength and current density satisfy the formula $\vec{J}=\sigma\vec{E}$. It can be seen that the electrostatic field is a special case of the steady electric field when the current density is $\vec{J}=\vec{0}$.The reason for these differences lies in the different field sources that generate electrostatic fields and steady electric fields. The electrostatic field is excited by static charges, while the steady electric field is co-excited by accumulated charges.

2.4 Comparative Study in the Presence Area[1]

When the charged conductor reaches the electrostatic equilibrium, the electrostatic field can only exist in the vacuum or dielectric outside the conductor. At this time, the conductor in the electrostatic field is an equipotential body, the surface of the conductor is an equipotential surface, and near the surface of the conductor, the electric field is perpendicular to the surface of the conductor. The steady electric field not only exists in vacuum and dielectric, but also can exist in non-ideal conductors. Therefore, there can be a potential difference between two points in the conductor. The surface of the conductor is not an equipotential surface. Near the surface of the conductor, the electric field is generally not perpendicular to the surface of the conductor. When the current flows from the ideal conductor into the general conductive medium, the current line is always perpendicular to the surface of the ideal conductor.

2.5 Comparative Study on Constitutive Equations of Media

In both dielectric and magnetic media, both the electrostatic field and the steady magnetic field must have physical quantities that describe the nature of a certain point in the field and the strength of the field. For electrostatic field, it can be described by electric field intensity $\vec{E}$ and electric flux density $\vec{D}$. For steady magnetic field, it can be described by magnetic flux density $\vec{B}$ and magnetic field intensity $\vec{H}$[1].

The electric field strength is the ratio of the electric field force received by a charge to the electric quantity of the charge: $\vec{E}=\vec{F}/q_0$. The magnetic induction intensity is the ratio of the maximum magnetic field force experienced by a moving charge to the product of the charge and the speed: $\vec{B}=\overrightarrow{F_{max}}/(q\vec{v})$. The relationship between electric field intensity and electric displacement vector: $\vec{D}=\varepsilon\vec{E}$ (In the dielectric), $\vec{D}=\varepsilon_0\vec{E}$ (In the vacuum). The relationship between magnetic field strength and magnetic induction: $\vec{B}=\mu\vec{H}$ (In the magnetic media), $\vec{B}=\mu_0\vec{H}$ (In the vacuum).

2.6 Comparative Study on Theorem

2.6.1 Starting From Electric and Magnetic Fields: (Vertical Connection)

In electrostatic field: Coulomb's law is the basic law of electrostatic field. Gauss's theorem and loop theorem can be derived from Coulomb's law and the principle of field superposition. In a steady magnetic field: Ampere's law is the basic law of a steady magnetic field. After learning the electrostatic field, Ampere's law can be derived directly by analogy to Coulomb's law. In addition, Gauss's theorem and Ampere's loop theorem can be derived from Biot-Savart's law.

Among them, the differential form of Gauss's theorem of electric (magnetic) field determines the divergence of electric (magnetic) field. The differential form of loop theorem of electric (magnetic) field determines the curl of electric (magnetic) field.

2.6.2 Starting From the Law: (Horizontal Connection)

The relationship between Coulomb's law and Biot-Savart law: Coulomb's law and Biot-Savart law are introduced from electrostatic field and steady magnetic field respectively, using the Lorentz coordinate transformation of special relativity[4] and charge invariance Knowledge, understanding the inherent unity of electric and magnetic fields.

For a relatively static charge, there is only the Coulomb force caused by the electrostatic field, and for the relatively moving charge, there is not only an electric field but also a magnetic field, that is, not only the Coulomb force but also the magnetic force. The magnetic field is caused by the movement of electric charges, and the transformation of the inertial system causes the magnetic field[4]. Electric and magnetic fields are closely linked through "movement." There is a deep connection between Coulomb's law and Biot-Savart's law, which shows that electromagnetic fields are indeed a unity[4].

With the continuous development and change of science, mankind's cognition of science is constantly improving and progressing, from one-sided to comprehensive, from partial to overall. The same is true for the understanding of electromagnetic theory: from point charge to generating electric field, current generating magnetic field; to the relationship between electric field $\vec{E}$ (or $\vec{D}$) and magnetic field $\vec{B}$ (or $\vec{H}$), forming a complete electromagnetic field theory, which has the following relationship[5]:

$$\vec{E}(\text{or}\,\vec{D}) \xrightarrow{\nabla\times\vec{E}=\frac{\partial\vec{B}}{\partial t}} \vec{B}(\text{or}\,\vec{H})$$

$$\nabla\cdot\vec{D}=\rho\uparrow_{\rho} \quad \frac{\partial\vec{D}}{\partial t}+\vec{j}=\nabla\times\vec{H}\uparrow_{\vec{j}} \quad \nabla\cdot\vec{B}=0$$

Fig. 1 Theoretical relationship diagram of electromagnetic field

From the relationship diagram, it can be seen that point charges and currents can generate electric and magnetic fields, and changing electric fields and changing magnetic fields can excite each other. The equal sign in the middle reveals the mutual transformation and interdependence between electricity and magnetism. They coexist in a unified In the electromagnetic field, the exchange and storage of energy occurs because of the mutual transformation between electricity and magnetism, as shown in Figure 2[6]:

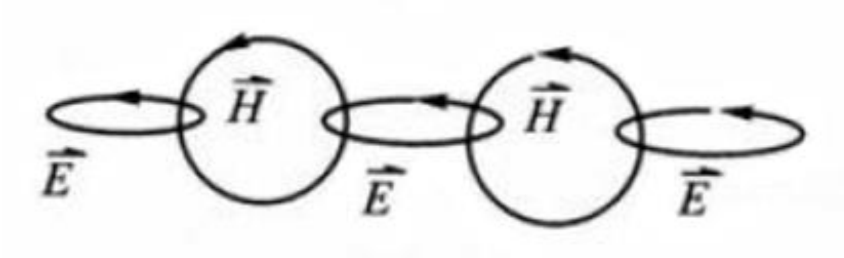

Fig. 2 Interconversion of electromagnetic energy

2.7 Comparative Study on Boundary Conditions

From the two basic theorems of electrostatic field[1]:

$$\oint_S \vec{D}\cdot \mathrm{d}\vec{S} = Q \tag{1}$$

and

$$\oint_l \vec{E}\cdot \mathrm{d}\vec{l} = 0 \tag{2}$$

$$\oint_S \vec{D}\cdot \mathrm{d}\vec{S}=\vec{D_1}\cdot\vec{n}\Delta S - \vec{D_2}\cdot\vec{n}\Delta S=\rho_s\Delta S \tag{3}$$

$$(\vec{D_1}-\vec{D_2}) = \rho_s \tag{4}$$

$$\vec{D} = \varepsilon\vec{E} \tag{5}$$

$$\varepsilon_1 E_{1n}-\varepsilon_2 E_{2n}=\rho_s \tag{6}$$

$$\oint_l \vec{E}\cdot \mathrm{d}\vec{l} = 0 \tag{7}$$

$$\oint_l \vec{E}\cdot \mathrm{d}\vec{l}=\vec{E_1}\cdot\vec{a_t}\Delta l-\vec{E_2}\cdot\vec{a_t}\Delta l=0 \tag{8}$$

$$\vec{a_t}\cdot(\vec{E_1}-\vec{E_2})=0 \tag{9}$$

To get the boundary condition expression:

$$\vec{n}\cdot(\vec{D_1}-\vec{D_2})=\rho_s \tag{10}$$

$$\vec{n}\times(\vec{E_1}-\vec{E_2})=0 \tag{11}$$

From the two basic theorems of steady electric field:

The same reasoning can be derived

$$\begin{cases} J_{1n}=J_{2n} \\ E_{1t}=E_{2t} \end{cases} \tag{12}$$

From the two basic theorems of steady magnetic field:

$$\oint_s \vec{B}\bullet \mathrm{d}\vec{S}=0 \tag{13}$$

and

$$\oint_L \vec{B}\bullet \mathrm{d}\vec{L}=\mu_0\sum_{(L内)} I \tag{14}$$

By analogy with the boundary conditions of the electrostatic field, the expression of the boundary conditions can be obtained:

$$\begin{cases} \vec{n}\cdot(\vec{B_1}-\vec{B_2})=0 \\ \vec{n}\times(\vec{H_1}-\vec{H_2})=\vec{J_S} \end{cases} \tag{15}$$

It can be seen that when the boundary conditions of the steady electric field and the electrostatic field are the same, the distribution of current density is exactly the same as that of electric field intensity. It can be seen that there is a certain correlation between the two, that is, the result of the electrostatic field can be used to directly solve the steady electric field; the same electric field as the electrostatic field can also be used to study the characteristics of the electrostatic field. This is the electrostatic analogy method.

Through the analysis of the law of the unity of opposites between the electrostatic field, the steady electric field and the steady magnetic field, the summarizes as shown in Table 1. The knowledge connection block diagram helps to improve memory efficiency, to understand the connections between existing ideas, to connect new ideas with existing knowledge, to develop understanding of knowledge, and to establish a structured knowledge network system.

Table 1 Comparison of Static Electric Field, Steady Electric Field and Steady Magnetic Field

	Electric Field		Magnetic Field
Field Source	Static Electric Field	Steady Electric Field	Steady Magnetic Field
Definition	Point charge q	Constant current I has an electric field in space. $I=\frac{Q}{t}$	Constant current produces a constant magnetic field.
Force	Coulomb's Law: $\vec{F}=\frac{kQ_1Q_2}{r^2}\vec{e_r}=\frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}\vec{e_r}$		Ampere's Law: $\vec{F_{12}}$ the force of current loop l_1 on l_2.
Variable	Electric field strength: $\vec{E}=\frac{U}{d}=\frac{Q}{4\pi\varepsilon_0 R^2},\vec{E}=-\nabla\phi$		Biot-Savart Law: $dB=k\frac{Idl\sin\theta}{r^2}$ ($k=\frac{\mu_0}{4\pi}$) (Electric field produces magnetic field) ↓elicit
Medium Constitutive Equation	Electric flux density: $\vec{D}=\varepsilon_0\vec{E}$	Current density: $\vec{J}=\frac{dI}{ds}$ or $\vec{J}=\frac{dI}{dl}$ $\vec{J}=\sigma\vec{E}$ (σ is conductivity)	Magnetic flux density: $\vec{B}=\mu_0\vec{H}$ ↓elicit Magnetic field strength: $\vec{H}$
	Method of calculating $\vec{E}$: 1.Direct integration. 2.Come first ϕ, Use $\vec{E}=-\nabla\phi$ to find $\vec{E}$. 3.Use Gauss theorem to find $\vec{E}$.		Method of calculating $\vec{B}$: 1.Direct integration. 2.Come first $\vec{A}$, Use $\vec{B}=\nabla\times\vec{A}$ to find $\vec{B}$. 3.Use Ampere's loop theorem to find $\vec{B}$.
Electric Potential	Electric potential ϕ: ϕ of point, line, surface, body.	Electromotive force ϕ: $\phi=\frac{\vec{P}\vec{a_r}}{4\pi\varepsilon_0 r^2}$	↑among them Magnetic vector position $\vec{A}$: $\vec{A}=\frac{\mu_0\vec{P_m}\times\vec{a_r}}{4\pi r^2}$
Two Basic Equations	1.Electric flux density $\vec{D}$ →Gauss Theorem: $\left.\begin{array}{l}\oint_S\vec{D}\bullet d\vec{S}=q\\ \vec{D}=\varepsilon_0\vec{E}\end{array}\right\}\Rightarrow\vec{E}$ $\nabla\bullet\vec{D}=\rho$ .Divergent Field. 2.Circulation of $\vec{E}$ → $\oint_l\vec{E}\bullet d\vec{l}=0$ $\nabla\times\vec{E}=\vec{0}$ .No Curl Field.	1.Current continuity equation → $\oint_S\vec{J}\bullet d\vec{S}=0$ $\nabla\bullet\vec{J}=0$. No Divergent Field. 2.Circulation of $\vec{E}$ → $\oint_l\vec{E}\bullet d\vec{l}=0$ $\nabla\times\vec{E}=\vec{0}$. No Curl Field.	1.Principle of Magnetic Flux Continuity: $\oint_S\vec{B}\bullet d\vec{S}=0$ $\nabla\bullet\vec{B}=0$. No Divergent Field. 2. $\vec{H}$ → Ampere Loop Theorem: $\oint_C\vec{H}\bullet d\vec{l}=I=\int_S\vec{J}\bullet d\vec{S}$ $\nabla\times\vec{H}=\vec{J}$. Curl Field.
Boundary Conditions	$\vec{n}\bullet(\vec{D}_2-\vec{D}_1)=\rho_S$ $\vec{n}\times(\vec{E}_2-\vec{E}_1)=0$	$\vec{n}\bullet(\vec{J}_2-\vec{J}_1)=0$ $\vec{n}\times(\vec{E}_2-\vec{E}_1)=0$	$\vec{n}\times(\vec{H}_2-\vec{H}_1)=\vec{J}_S$ $\vec{n}\bullet(\vec{B}_2-\vec{B}_1)=0$

3. Conclusion

Traditional "Electromagnetics" study usually starts with electrostatic field and steady electric field, and then steady magnetic field. The knowledge between knowledge points is isolated from each other, cannot have a horizontal and comprehensive understanding of the electromagnetic field, and the input and output effects are not ideal. This article adopts the method of comparative learning. By comparing the electric field and the magnetic field vertically and horizontally, it is helpful to deepen the understanding of the essence of knowledge, experience the connection between knowledge, and construct the scattered knowledge points into a knowledge network, which is helpful for learning knowledge.

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References

[1] H.P. Guo, X.G. Liu: Electromagnetic Field and Electromagnetic Wave (Xi'an Electronic Science and Technology University Press, China 2017).(In Chinese).

[2] Maxwell: General Theory of Electromagnetism (Peking University Press, China 2010).

[3] Y.P. Wang: Engineering Electrodynamics (Xi'an Electronic Science and Technology University Press, China 2007).(In Chinese).

[4] Z.P. Yin: Coulomb's law and Biot-Savart's law, Journal of Railway Teachers College,Vol. 13 (1996) No.1, p.53-56.

[5] G.D. Wang, L.Z. Wang, C.Y. Zheng, et al: Unity of electromagnetic field, Journal of Jilin Normal University,Vol. 28(2007) No.1, p.115-116.

[6] C.H. Liang, Y.J. Xie, B.R. Guan: Concise Microwave (Higher Education Press, China 2006).(In Chinese).