

Analysis of Strip Foundation Settlement of Buildings on Adjacent Caused by Tunnel Excavation

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Abstract

This article in view of the tunnel excavation form buildings inside the settling tank below the upper structure, lower base and foundation interaction of three factors, establish a synergy with these three factors model, differential equation is derived to study settlement. Put forward the discriminant method of building security, and impact on the tunnel excavation within the scope of areas for research. Combination with the engineering calculation of verification for future influence within the scope of the structure of the underground tunnel excavation safety protection provides a certain reference and reference.

Keywords

Tunneling; Differential equation; Safety criterion; Interaction.

1. Introduction

In order to alleviate the increasingly serious urban traffic congestion, more and more cities choose fast and efficient subway as the main carrier of public transport. In the process of subway construction, tunnels are often near existing buildings. Because tunnel excavation causes settlement deformation of adjacent buildings and even destroys them, the influence of tunnel excavation on settlement of adjacent existing buildings has become a widespread concern.

Previous studies on the impact of tunnel excavation on adjacent buildings mainly focus on the superstructure, but the interaction between foundation and soil has not been considered in depth [1]. In engineering practice, it is affected by the synergy of three factors: superstructure, foundation and foundation. Duan Jingming used the surface subsidence model of mining area to analyze. In this paper, the combination of the three is analyzed. According to the commonly used PECK surface settlement formula, the shear bending beam model of upper frame structure with shear on elastic foundation and bent strip foundation is established, and the settlement and its influence under the synergistic action are deduced [2].

2. Establishment of mechanical model

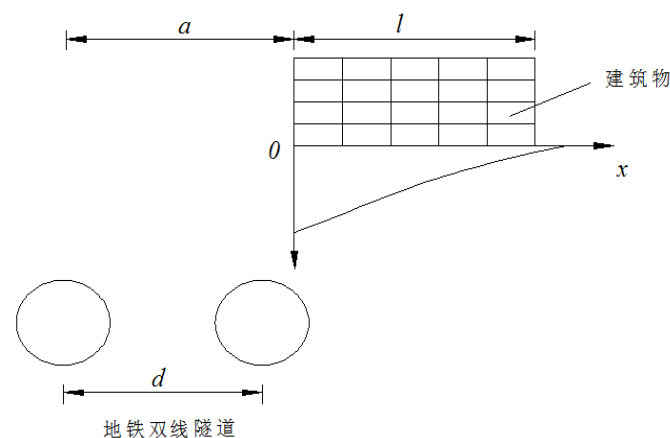


Fig. 1 Building Settlement Coordinate System Based on Ground Subsidence

With the continuous progress of tunnel excavation, buildings located in different locations of settlement trough are affected by different changes. The interaction between the upper structure, the lower foundation and the lower foundation of the building is complex. In order to study the synergistic action mechanism of the three, the coordinate system of building settlement based on ground subsidence is established, and the formula deduction and the example calculation are carried out to analyze [3].

2.1 Establish conditions

Combining with the characteristics of synergistic forces, the model is illustrated in Fig. 2.

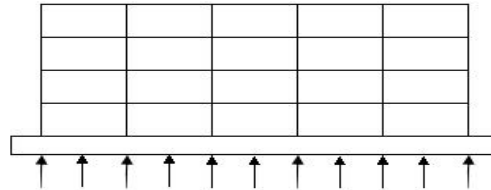


Figure 2 Computing Model

(1) The PECK model is chosen for land subsidence. In this coordinate system, the formula of land subsidence curve is as follows:

$$S(x) = s_{\max} e^{-\frac{(x+a)^2}{2i^2}} + s_{\max} e^{-\frac{(x+a-d)^2}{2i^2}} \tag{1}$$

$$s_{\max} = \frac{V_i}{i\sqrt{2\pi}} \approx \frac{V_i}{2.5i} \tag{2}$$

Among them: $S(x)$ —The ground subsidence from the central axis of the left tunnel $a+x$ is taken as $0 \leq x \leq l$ in this model;

V_i —Ground Loss per Unit Length of Tunnel Caused by Excavation;

s_{\max} —Maximum Surface Settlement at the Center of Tunnel;

i —Width coefficient of surface subsidence trough.

(2) According to the knowledge of basic engineering, the foundation soil is linear elastomer, which conforms to Winkler elastic foundation model. According to Winkler's theory, there is a linear relationship between the reaction force at any point in the foundation and the relative settlement of the building at that point. [4], Namely:

$$P(x) = k [W(x) - S(x)] \tag{3}$$

In the above formula: $S(x)$ —Ground subsidence corresponding to any x in a building;

$W(x)$ —Settlement at x from the left end of the building;

k —Machine Tool Coefficient of Foundation;

$P(x)$ —Reaction at any point in the foundation beneath the building.

(3) The upper frame structure of the building suffers from shear failure, while the lower strip foundation suffers bending failure. [5]. When they are combined into shear-bending beams on Winkler elastic foundation, their deflection differential equations [6] are as follows:

$$EJ \frac{d^4W(x)}{dx^4} - (GF + g) \frac{d^2W(x)}{dx^2} = q(x) - p(x) \tag{4}$$

In the formula: EJ —Bending stiffness of beam;
 GF —Shear stiffness of frame;
 q —Vertical Load Transferred from Frame Structures;
 $p(x)$ —Foundation reaction force;
 g —Restraint Line Stiffness of First-floor Column Bottom.

2.2 Set up equation

By connecting (1), (3), (4) three formulas, we can get the following formulas:

$$EJ \frac{d^4W(x)}{dx^4} + R \frac{d^2W(x)}{dx^2} + kW(x) = q + ks_{\max} e^{-\frac{x^2}{2l^2}} + ks_{\max} e^{-\frac{(x+d)^2}{2l^2}} \tag{5}$$

In the formula $R = -(GF+g)$

The upper one is the settlement trough formed by tunnel excavation, the upper one is the frame structure, and the lower one is the differential equation of synergistic action of strip foundation buildings.

2.3 Equation solving

(1) The solution of equation (5) corresponds to the solution of homogeneous equation (that is, the right end of equation is 0):

Given: $w = e^{\lambda x}$, $b = \frac{R}{EJ}$, $n = \frac{k}{EJ}$, with the corresponding homogeneous equation, there are the following equations:

$$\lambda^4 + b\lambda^2 + n = 0 \tag{6}$$

Get the solution:

$$\lambda_{1,2} = \pm \frac{1}{2} \sqrt{-2b + 2\sqrt{(b^2 - 4n)}}$$

$$\lambda_{1,2} = \pm \frac{1}{2} \sqrt{-2b - 2\sqrt{(b^2 - 4n)}}$$

Then the solution of the homogeneous equation corresponding to equation (5) is obtained as follows:

$$w(x) = c_1 e^{d_1 x} + c_2 e^{-d_1 x} + c_3 e^{d_2 x} + c_4 e^{-d_2 x}$$

$$d_1 = \pm \frac{1}{2} \sqrt{-2b + 2\sqrt{(b^2 - 4n)}} \tag{7}$$

$$d_2 = \pm \frac{1}{2} \sqrt{-2b - 2\sqrt{(b^2 - 4n)}}$$

In the formula, C_1 、 C_2 、 C_3 and C_4 are undetermined coefficients.

(2) Solving the Special Solution of the Primitive Equation

According to the superposition principle of special solutions of differential equations, the special solutions of three independent equations are obtained respectively, and then the three solutions are superposed to obtain the special solutions of equation .

$$EJ \frac{d^4W(x)}{dx^4} + R \frac{d^2W(x)}{dx^2} + kW(x) = ks_{\max} e^{-\frac{x^2}{2l^2}} \tag{8}$$

$$EJ \frac{d^4W(x)}{dx^4} + R \frac{d^2W(x)}{dx^2} + kW(x) = q \tag{9}$$

$$EJ \frac{d^4 W(x)}{dx^4} + R \frac{d^2 W(x)}{dx^2} + kW(x) = kS_{max} e^{-\frac{(x+d)^2}{2t^2}} \tag{10}$$

Solve the special Solutions to Equation (8)

According to higher mathematics, the method of solving non-homogeneous differential equation with constant coefficients satisfies $e^{\lambda x} p_m(x)$ form. In this paper, the equation (4.8), $t = 0$ $p_m(x) = q$. Then equation (8) has the following special solutions

$$w_1^* = x^u q_m(x) e^{\lambda x} \tag{11}$$

Among them, $q_m(x)$ is a polynomial of the same number as $p_m(x)$, u is the characteristic equation of the original equation, that is, the number of repetitions containing root λ in equation (6). In this equation (8), because the right end of the equation is q , so $\lambda = 0$. However, there is no $\lambda = 0$ in the root of the eigenvalue equation (6) corresponding to equation (8) in this paper, so $u = 0$ (because there is no repetition with the root of the eigenvalue equation), so the special solution of equation (8) in this paper is as follows:

Solve the Special Solutions to Equation (9)

$$w_1^* = q_m(x) = q/k \tag{12}$$

It is assumed that the special solution is in the form of:

$$w_2^*(x) = fe^{-bx^2} \tag{13}$$

Solving parameters f and b in equation (9) of this paper.

Order, then equation (9) is converted to:

$$EJ \frac{d^4 W(x)}{dx^4} + R \frac{d^2 W(x)}{dx^2} + kW(x) = kS_{max} e^{-mx^2} \tag{14}$$

Take λe^{-bx^2} into equation (14):

$$EJ \left[-2bf(-6b + 24b^2x^2 - 8b^3x^4) \right] e^{-bx^2} - 2bRf(1 - 2bx^2) e^{-bx^2} + kfe^{-bx^2} = kS_{max} e^{-mx^2} \tag{15}$$

Divide the two sides of the equation by λe^{-bx^2} at the same time, obtain:

$$EJ \left[-2b(-6b + 24b^2x^2 - 8b^3x^4) \right] - 2bR(1 - 2bx^2) + k = \frac{kS_{max}}{f} e^{bx^2 - mx^2} \tag{16}$$

By expanding the power series on the right side of the equation, obtain:

$$EJ \left[-2b(-6b + 24b^2x^2 - 8b^3x^4) \right] - 2bR(1 - 2bx^2) + k = \frac{kS_{max}}{f} \left[1 + x^2(b - m) + \frac{1}{2}x^4(b - m)^2 \right] \tag{17}$$

From the coefficients of corresponding terms at both ends of the equation are equal, the following can be obtained:

$$\begin{cases} 16EJb^4 = \frac{kS_{max}}{2f}(b - m)^2 \\ -48EJb^3 + 4Rb^2 = \frac{kS_{max}}{f}(b - m) \\ 12EJb^2 - 2Rb + k = \frac{kS_{max}}{f} \end{cases} \tag{18}$$

Solve the equations, obtain:

$$\begin{cases} b = \frac{R + \frac{6EJ}{i^2} + \sqrt{\left(R + \frac{6EJ}{i^2}\right)^2 - \frac{40REJ}{i^2}}}{40EJ} \\ f = \frac{k_s}{k + \frac{3\left(R + \frac{6EJ}{i^2} \sqrt{\left(R + \frac{6EJ}{i^2}\right)^2 - \frac{40REJ}{i^2}}\right)^2}{400EJ} - R \frac{\left(R + \frac{6EJ}{i^2} \sqrt{\left(R + \frac{6EJ}{i^2}\right)^2 - \frac{40REJ}{i^2}}\right)^2}{20EJ}} \end{cases} \quad (19)$$

3 seek special Solutions to Equation (10)

It is assumed that the special solution is in the form of:

$$w_3^* = he^{-j(x+d)^2} \quad (20)$$

Solving parameters h and j in equation (10) of this pape, Obtain: $he^{-j(x+d)^2}$, order $x + d = t$,

Because the solution is similar to the solution of $w_2^*(x) = fe^{-bx^2}$ in equation (10), it will not be repeated here. By solving the problem, we can get that $h = f$, $j = b$.

By superposing the special solutions of the above three equations, the special solutions of the original equation are obtained as follows:

(3) Solving the General Solution of the Primitive Equation

According to the solution of differential equation, the general solution of differential equation (5) is the sum of the solution and the solution of its corresponding homogeneous equation.

(3) Solving undetermined coefficients c_1 、 c_2 、 c_3 、 c_4

By solving the second and third derivatives of the above formula, the following results are obtained:

$$\frac{d^2W(x)}{dx^2} = c_1d_1^2e^{d_1x} + c_2d_1^2e^{-d_1x} + c_3d_1^2e^{d_2x} + c_4d_2^2e^{-d_2x} - w'' \quad (21)$$

$$\frac{d^3W(x)}{dx^3} = c_1d_1^3e^{d_1x} - c_2d_1^3e^{-d_1x} + c_3d_1^3e^{d_2x} - c_4d_2^3e^{-d_2x} - w''' \quad (22)$$

By using the boundary conditions, the equations can be obtained.

$$\begin{aligned} c_1d_1^2 + c_2d_1^2 + c_3d_2^2 + c_4d_2^2 - w''(x=0) &= 0 \\ c_1d_1^2e^{d_1l} + c_2d_1^2e^{-d_1l} + c_3d_2^2e^{d_2l} + c_4d_2^2e^{-d_2l} - w''(x=l) &= 0 \\ c_1d_1^3 - c_2d_1^3 + c_3d_2^3 - c_4d_2^3 - w'''(x=0) &= 0 \\ c_1d_1^3e^{d_1l} - c_2d_1^3e^{-d_1l} + c_3d_2^3e^{d_2l} - c_4d_2^3e^{-d_2l} - w'''(x=l) &= 0 \end{aligned} \quad (23)$$

The above four equations are combined to get the equations, and the coefficients c_1 , c_2 , c_3 and c_4 are solved. The above four coefficients are brought into equation (23) to obtain the settlement formula of the building under the synergistic action.

The additional bending moment of strip foundation is:

$$M(x) = EJ \frac{d^2}{dx^2} W(x) \quad (24)$$

The additional shear force of strip foundation is as follows:

$$Q(x) = EJ \frac{d^3}{dx^3} W(x) \tag{25}$$

3. Parameter analysis

Taking the engineering geology of Jixiangcun Station to Taibai District Station of Xi'an Metro Line 3 as an example, this paper studies the influence of different factors on pipeline deformation.

3.1 Influence of distance between buildings and tunnels

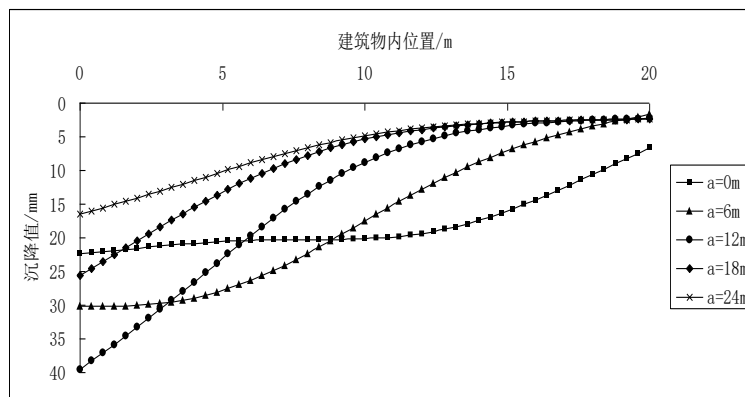


Figure 3 Settlement curves of buildings with different a values

Considering the relative position between the building and the tunnel, the distance a between the left end of the building and the middle axis of the left line is 0 m (the left end of the building is above the left line), 6 m (the left end of the building is in the middle of the two lines), 12 m (the left end of the building is above the right line), 18 m (the left end of the building is 6 m from the right line), 24 m (the left end of the building is 12 m from the right line). From Figure 3, it is found that the maximum settlement of a building increases first and then decreases as the distance between the left end of the building and the center axis of the left line increases. The corresponding maximum values are 22.3mm, 30.2mm, 39.5mm, 25.5mm and 16.5mm, respectively. The same is true for the variation of uneven settlement. When a = 12m, that is, the left end of the building is directly above the right line, the uneven settlement of the building is the largest, and the peak settlement is the largest.

From Figure 4, the maximum additional bending moment of strip foundation is - 65.1KN * m, - 84.5KN * m, 86.8KN * m, 54.3KN * m and 32.4KN * m, respectively, when the distance between the left end of the building and the center axis of the left line is different.

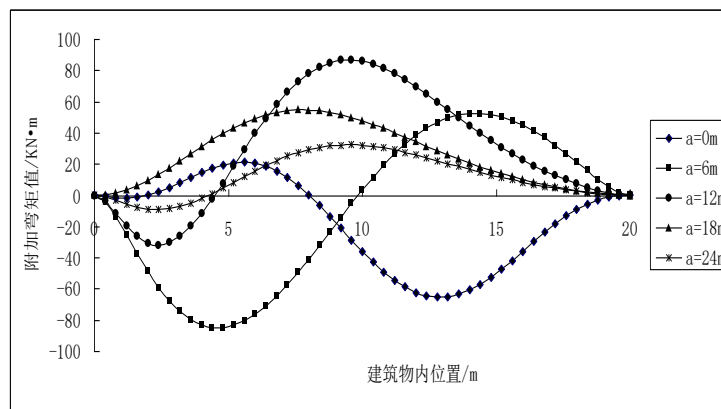


Fig. 4 Additional Moment of Foundation under Different A Values

When a = 6m, that is, the left end of the building is located in the middle of the two lines, the distribution of additional bending moment of strip foundation is generally in the form of central

symmetry centered on the midpoint of the building, and the value of additional bending moment varies most. When $a = 12\text{m}$, that is, the left end of the building is above the right line, the peak value of additional moment is the largest.

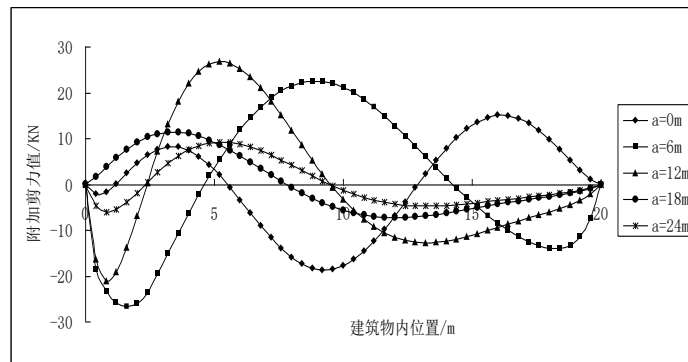


Fig. 5 Shear force of foundation accessories under different a values

From Fig. 5, the maximum additional shear force of strip foundation is -18.7KN , -26.9KN , 28.2KN , 11.4KN and 9.3KN respectively when the distance a between the left end of the building and the center axis of the left line is different. When $a = 12\text{m}$, that is, the left end of the building is directly above the right line, the peak additional shear force of the strip foundation is the largest.

From the above three curves, when the edge of the building is directly above the right line, the settlement of the building and the internal force of the foundation change the most.

3.2 Influence of tunnel section size

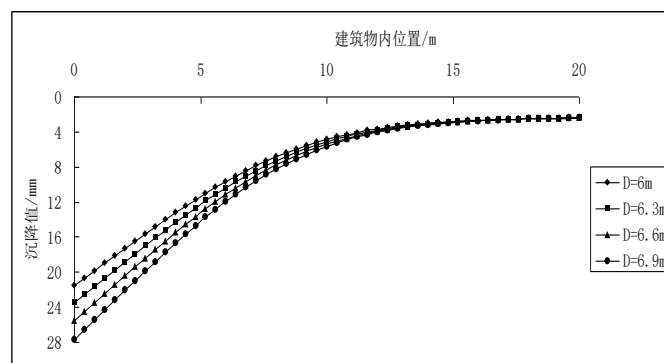


Figure 6 Settlement of Different D Buildings

The common section of metro tunnel is circular, so the tunnel section is circular, and the diameter is 6m , 6.3m , 6.6m and 6.9m , respectively, for calculation and analysis.

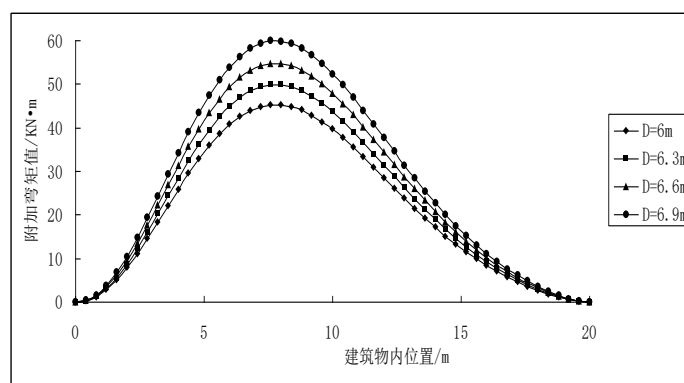


Figure 7 Additional Moments of Different D Foundations

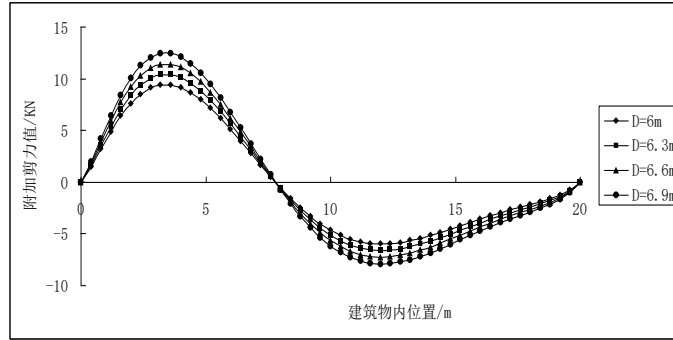


Figure 8 Additional Moments of Different D Foundations

From the above three drawings, the maximum settlement of the corresponding building is 21.5m m, 23.4mm, 25.5mm and 27.7mm, and the corresponding additional bending moments are 59.9KN * m, 54.8KN * m, 49.9KN * m and 45.2KN * M. The additional shear forces are 9.5KN, 10.4KN, 11.4KN and 12.5KN, respectively. The corresponding settlement and the maximum position of additional bending moment and additional shear force are in the same position. The value of influencing factor D increases with the law of equal difference, and the additional bending moment and shear force of the foundation obtained by calculation increase accordingly, and the increase increases gradually. With the increase of tunnel cross-section size, the maximum value of surface settlement increases, and the width of settlement trough becomes wider, which increases the damage to the upper buildings. Therefore, in the case of engineering conditions permitting, reducing the tunnel cross-section size is conducive to the protection of buildings.

3.3 Influences of Foundation Stiffness

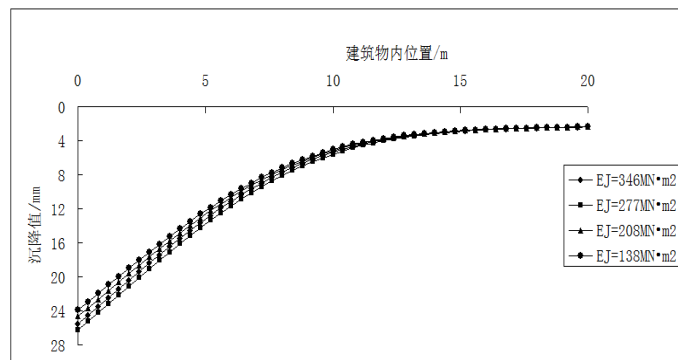


Figure 9 Settlement of different EJ buildings

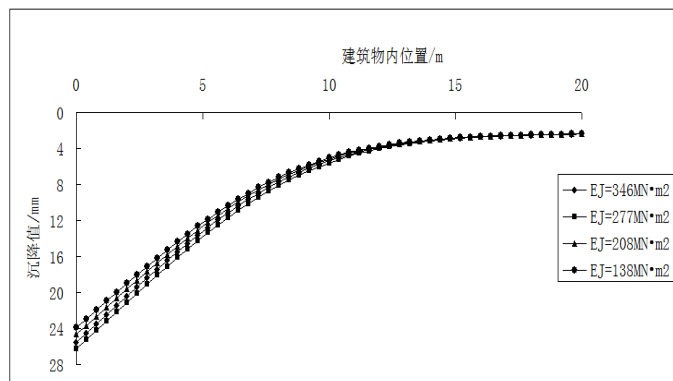


Figure 10 Additional Moments of Different EJ Foundations

Foundation is directly affected by construction. Under the condition that other parameters remain unchanged, the flexural stiffness of the foundation is 138MN m2, 208 MN m2, 277MN m2 and

346MN m², respectively, and is brought into the deduced analytical formula for calculation and analysis.

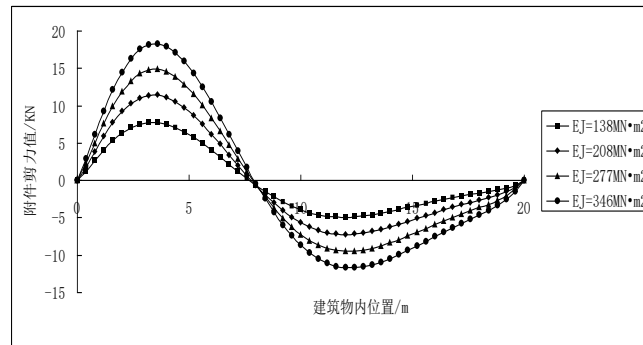


Figure 11 Additional Shear Forces of Different EJ Foundations

From the above three figures, the maximum settlement corresponding to the increase of foundation stiffness is 23.9 mm, 24.7 mm, 25.5 mm and 26.6 mm, respectively. The corresponding maximum additional bending moments are 36.9KN * m, 54.7KN * m, 72.4KN * m and 89.6KN * M. The corresponding maximum additional shear forces are 7.7KN, 11.4KN, 14.9KN and 18.4KN, respectively. The stiffness of the foundation increases by an equal margin, while the increment of additional bending moment and additional shear force obtained by calculation decreases gradually [4]. The maximum additional bending moment and additional shear force are obtained at the same position. With the increase of EJ, the resistance of structure to surface deformation is stronger, so it is of practical significance to increase the stiffness of foundation in design and reinforcement.

3.4 The influence of building length

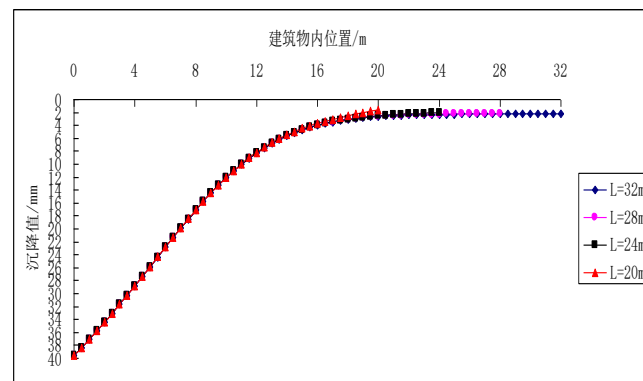


Figure 12 Settlement of Buildings of Different Length

In order to explore the rule of building length l , the length of building is 20m, 24m, 28m and 32m, respectively, which are brought into the deduced analytical formula for calculation and analysis.

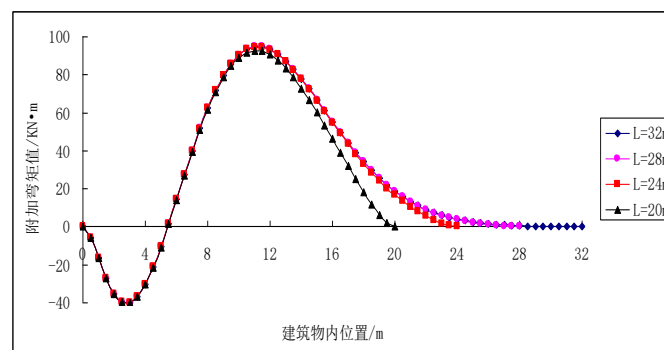


Fig. 13 Additional bending moments of foundations of different lengths

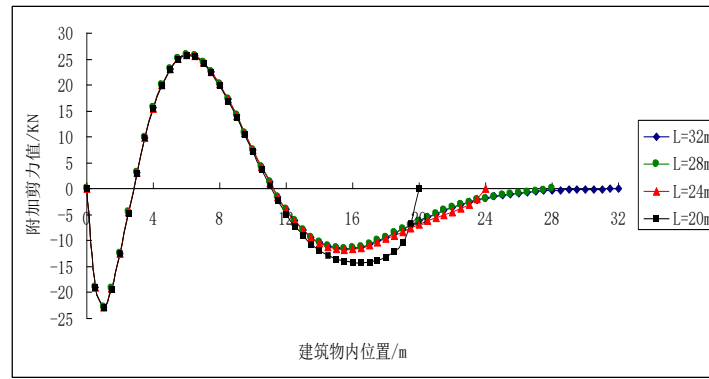


Fig. 14 Additional Shear Force of Foundation with Different Length

From figs. 12 to 14, it is found that the settlement curves of buildings almost coincide. With the increase of l , the settlement curves gradually extend, and the additional bending moments and shear forces of strip foundations change roughly as well.

3.5 Influence of tunnel spacing

The common double-track tunnels in metro tunnels are mostly two parallel single-track tunnels. In order to reduce the interaction between the two tunnels, one of them is usually excavated first, and the other is excavated after the advancing distance of the face reaches or exceeds the influencing range. So their geological conditions and construction methods are basically the same. Therefore, the two parallel lines can be separately constructed and superimposed to obtain the ground subsidence^[3] caused by the double-line construction. Accordingly, the set spacing d is 10m, 12m, 14m and 16m, respectively, and the deduced analytical formula is introduced for calculation and analysis.

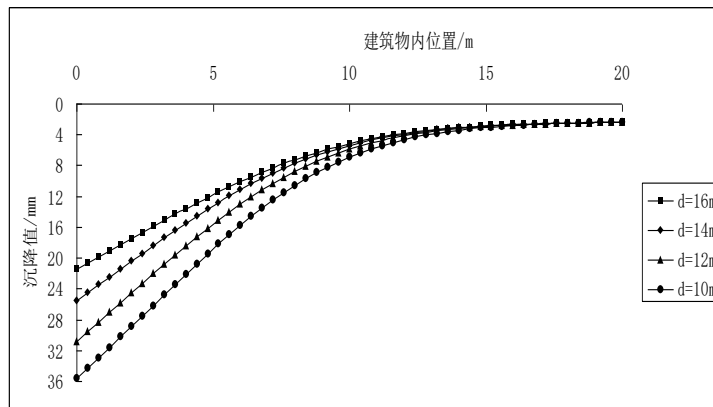


Fig. 15 Settlement of buildings in different d

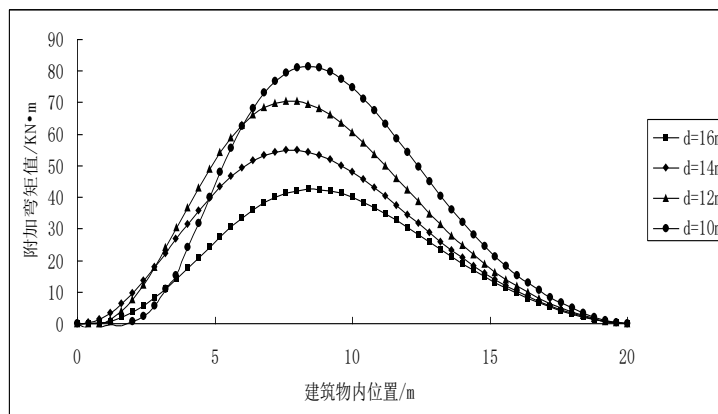


Fig. 16 Additional bending moments of different d foundations

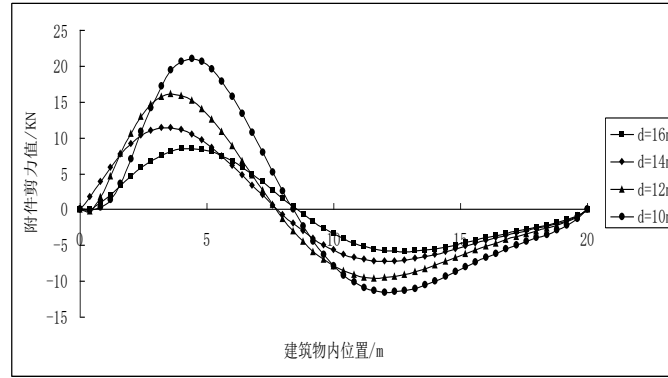


Figure 17 Additional shear force of different d foundations

From Fig. 15 to Fig. 17, the corresponding deformation increases with the decrease of D. The corresponding maximum settlement is 21.4 mm, 25.5 mm, 30.8 mm and 35.6 mm, respectively. The corresponding maximum additional bending moments are 42.5KN m, 54.7KN m, 70.4KN m and 81.5KN m, respectively. The corresponding maximum additional bending moments are 8.5KN, 11.4KN, 16.1KN and 21.0KN, respectively. The corresponding settlement is in the same position, and the position of the maximum additional bending moment and shear force is C-shaped vertically. In a certain range, the value of influencing factor D increases with the law of equal difference, and the additional bending moment and additional shear force of the foundation obtained by calculation decreases accordingly, but the amount of reduction increases gradually.

4. Building Safety Analysis

When the bending of strip foundation is considered as a double-ribbed rectangular section bending member, the ultimate bending moment is as follows:

$$M_{max} = \alpha_s \alpha_1 f_c b h_0^2 \tag{26}$$

In the formula, α_s —Flare Resistance Coefficient of Cross Section

α_1 —Environmental classification

b —Foundation width

h_0 —Placement Height of Concrete Reinforcement Bars

f_c —Axial compressive strength of concrete

The example parameters are introduced into the formula in turn, and the following conclusions are drawn:

$$\begin{aligned} M_{max} &= \alpha_s \alpha_1 f_c b h_0^2 = \\ &0.399 \times 1.0 \times 14.3 \times 1.3 \times 340^2 \\ &= 857.5 \text{KN} \cdot \text{m} \end{aligned}$$

Using the inverted beam method to solve the original bending moment of strip foundation, the following conclusions are drawn:

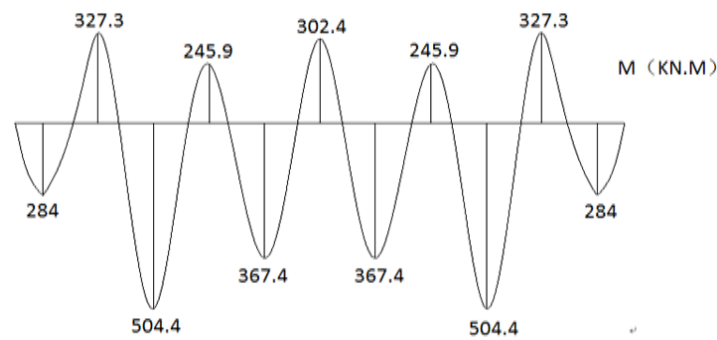


Fig. 18 The original bending moment diagram of strip foundation

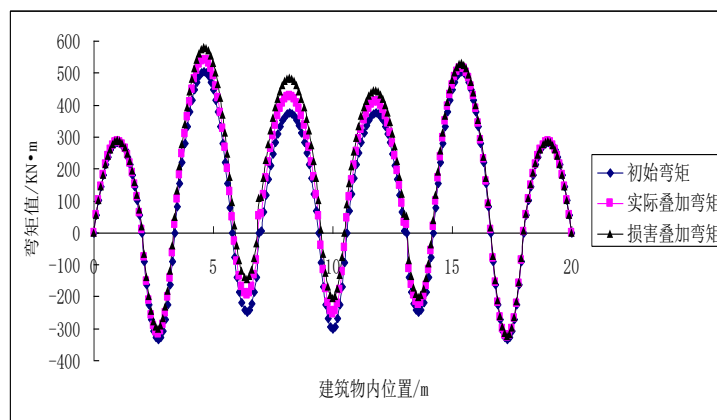


Fig. 19 Superimposed Bending Moment Diagram of Bar Foundation

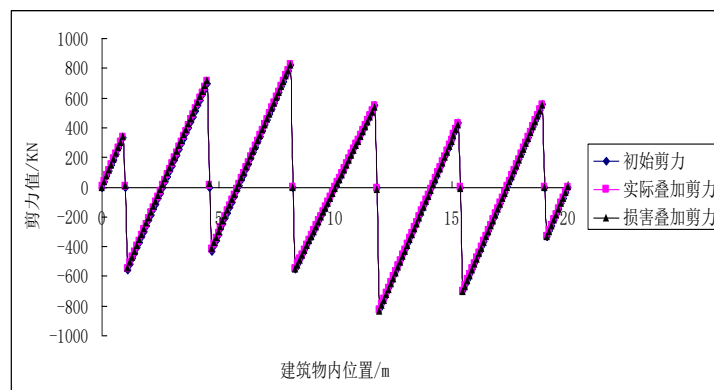


Figure 20 Superposition Moment Diagram of Bar Foundation

From the above figure, it can be concluded that the comprehensive bending moment obtained by superposition of additional bending moment and initial bending moment and the damage comprehensive bending moment calculated by taking the unfavorable value of influencing parameters are all within the limit bearing moment range of the foundation, so the building foundation can continue to be used, but prevention and cure need to be considered.

5. Conclusion

(1) In this paper, a shear-bending beam model of upper frame structure with shear on elastic foundation and bent strip foundation under bending is established. The differential equation of synergistic action is deduced, and the additional settlement of building and the additional bending and shear force of appendage of foundation are solved.

(2) Taking the engineering geology of the section between Taibai District Station and Jixianglu Road Station of Xi'an Metro Line 3 as an example, this paper makes parameter analysis of displacement equation, and draws the following conclusions and studies the sub-regions within the influence area of tunnel excavation.

(3) Taking the engineering geology of the section between Taibai District Station and Jixianglu Road Station of Xi'an Metro Line 3 as an example, the formulas deduced above are analyzed in detail.

(4) In this paper, a method for judging the safety of buildings is proposed, which superimposes the additional bending moment of foundation and the original existing bending moment, and then compares it with the ultimate bearing moment of foundation. By taking additional bending moments under unfavorable parameters into superposition, the safety condition of buildings can be judged.

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