Bank Loan Problem based on Nonlinear Programming

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Abstract

In practice, due to the scale of micro, small and medium enterprises are relatively small, also the lack of mortgage assets. So banks are usually on the basis of credit policy, enterprise trade instrument information and the influence of the upstream and downstream enterprises, the strength, stable supply and demand of the enterprise to provide loans, and can the high credibility, the credit risk of small business enterprise give preferential interest rates. Commercial loans are usually divided into three categories according to the degree of risk: normal, secondary and suspicious. The line and interest rate of these three categories are decided by nonlinear programming aimed at maximizing the bank's income. The range of interest rate and interest rate can be obtained when the annual total credit amount is within a certain range.

Keywords

Small and Medium-sized Enterprises; Credit Risk; Nonlinear Programming.

1. Introduction

The bank evaluates the credit risk of small, medium and micro enterprises based on their strength and reputation, and then determines whether to lend money and credit strategies such as loan line, interest rate and term according to credit risk and other factors. The existing bank to determine the loan line of enterprises to lend 100 million to 1 million yuan. The annual interest rate is 4 to 15 percent. The term of the loan is one year. According to the credit rating of the enterprise is divided into A, B, C three levels. Now it is necessary to conduct a quantitative analysis on the credit risks of 123 small and medium-sized enterprises, and give the bank's credit strategy for these enterprises when the annual total credit is fixed. When the total amount of bank's annual credit is fixed, the decision on the amount and interest rate is made by nonlinear programming with the goal of maximizing the bank's income.

2. Credit strategy formulation

2.1 Nonlinear programming based on bank income maximization

2.1.1 The determination of the objective function

Assume that each company in the normal class, the secondary class, and the suspect class has a credit limit of k_1, k_2, k_3. The interest rates for three categories are r_1, r_2, r_3.The number of companies is a_{ij}. Among them, i represents the i-th category, 1, 2, and 3 respectively represent the normal category, the sub-category, and the suspicious category; j represents the company's reputation level, A represents the first category, B represents the second category, and C represents the third category. That is, a_{ij} is the number of companies in the i-th category and the j-type reputation level.

Thus, the objective function for banks to achieve maximum benefits can be obtained as follows:

$$max\sum_{i=1}^{3}\sum_{j=1}^{3}r_i k_i a_{ij}-\sum_{i=1}^{3}(k_i\rho_i\sum_{j=1}^{3}a_{ij})$$

2.1.2 Establishment of constraints

(I) Determination of variable range

According to the conditions of the bank to determine the loan enterprise loan line is ten thousand yuan. The annual interest rate is 4% ~ 15%, and the range of r and k can be obtained as follows:

$$0.04 \leq r_1, r_2, r_3 \leq 0.15$$

$$10 \leq k_1, k_2, k_3 \leq 100$$

(II) Loan portfolio interest rate constraint

Since banks have different credit degrees for normal classes, secondary classes and suspicious classes, we consider to set a range of interest rates, that is $r_1 < r_2 < r_3$.

(III) Introducing upper bound restrictions in banking supervision [1]

The banking laws and financial authorities of all countries have strict regulations on asset liability management. For Chinese banking industry, these regulations mainly restrict the upper limit: the total loan of a single company or individual shall not exceed 10% of the bank's capital.

$$\mathbf{0 \leq \frac{k}{L} \leq 10\%}$$

$$k = \text{random}\ \{k_1, k_2, k_3\}$$

Where L represents the total annual bank credit, k refers to the loan line of a single company in each category, and $\frac{k}{L}$ represents the percentage of the total loan of the company that does not exceed the bank's capital. The advantage of this restriction is to prevent loans from being concentrated on a certain customer, thereby avoiding credit risk.

$$\mathbf{0 \leq \frac{y}{L} \leq 60\%}$$

$$y = 10 \cdot max\{x_1, x_2, x_3\}$$

Where x_1 represents the total value of the company's price and tax, x_2 represents the profit, and x_3 represents the tax rate. The three are the overall strength of the company. And y represents the total loan amount of the largest 10 customers in all companies accounted for the total loan amount of the bank, and $\frac{y}{L}$ represents the percentage of the total loan amount of the largest 10 customers to the total loan amount of the bank. The advantage of this restriction is to avoid over-concentration of loans and control portfolio risk.

(IV) The magnitude of a_{ij}

According to the data of the number of enterprises in each category, we have calculated the value of a_{ij} as shown in the figure below:

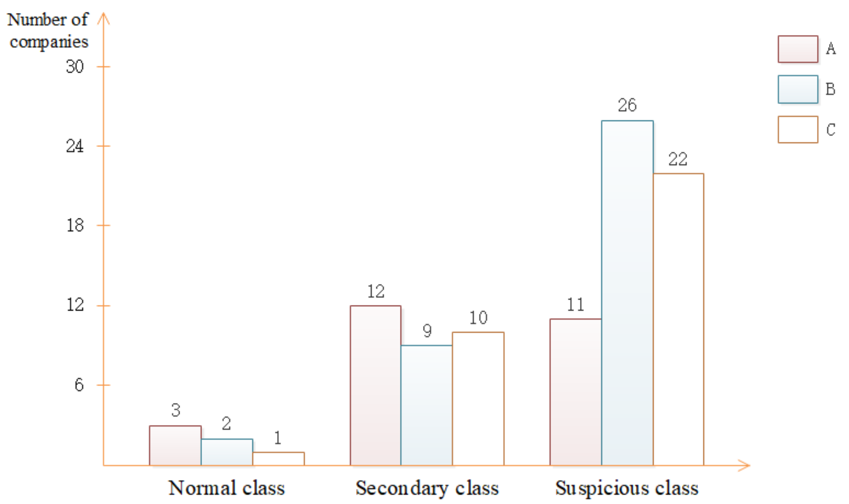
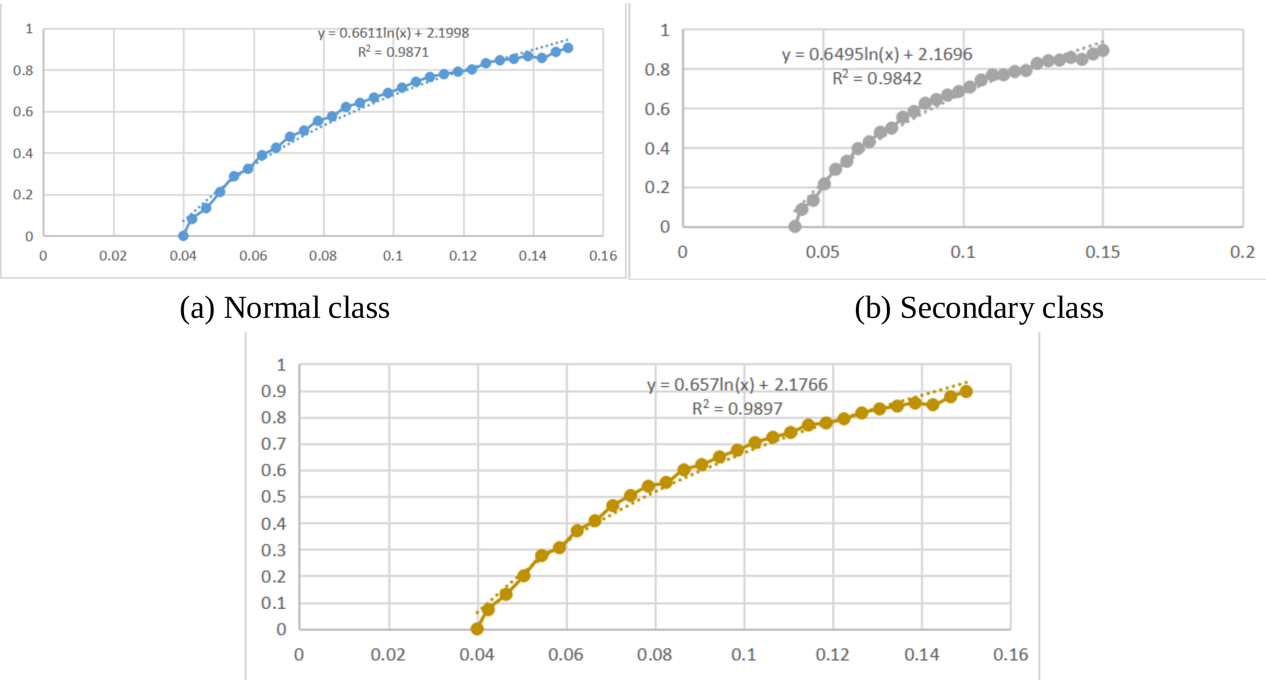


Figure 1. The number of companies at different levels under different categories

(V) The relationship between the interest rate r and the total loss rate ρ_i

$$\rho_i = \sum_{i=1}^{3} v_i \lambda_i$$

Where v_i is the proportion of the number of companies with the i-th reputation level in different categories to the number of headquarter companies in that category, and λ_i is the customer churn rate under the i-th reputation level. We have obtained a series of data on the total customer churn rate of different categories under different interest rates, and used Excel to fit them to the function. The results are shown in the following figure:



(a) Normal class (b) Secondary class

(c) Suspicious class

Figure 2. The relationship between churn rate and interest rates

It can be seen from the figure that the functional relationship between various customer churn rates and interest rates is: normal class $\rho_1 = 0.6611\, lnr_1 + 2.1998$, secondary class $\rho_2 = 0.6603\, lnr_2 + 2.1923$, suspicious class $\rho_3 = 0.657\, lnr_3 + 2.1766$.

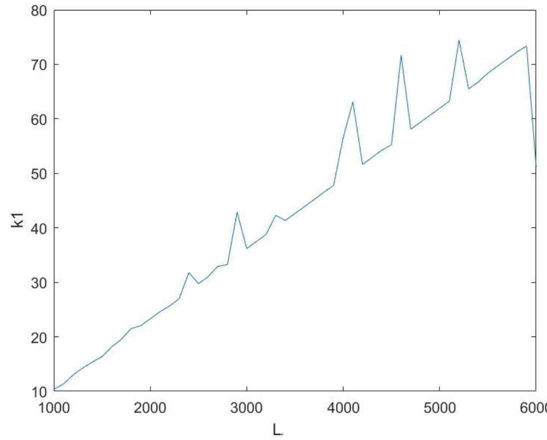
In summary, the nonlinear programming is:

$$max \sum_{i=1}^{3}\sum_{j=1}^{3} r_i k_i a_{ij} - \sum_{i=1}^{3}\left(k_i \rho_i \sum_{j=1}^{3} a_{ij}\right)$$

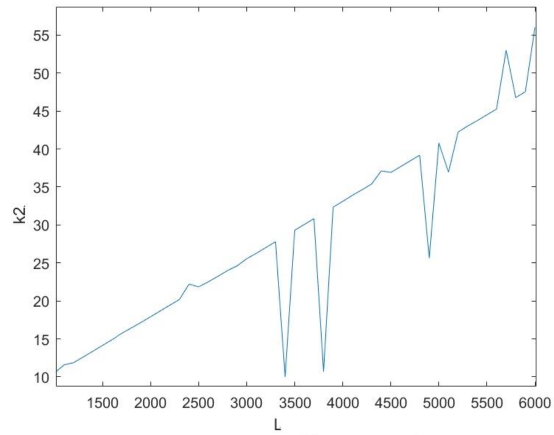
$$\begin{cases} 0.04 \le r_1 \le r_2 \le r_3 \le 0.15 \\ 10 \le k_1, k_2, k_3 \le 100 \\ 0 \le \dfrac{k_1}{L} \le 10\%, 0 \le \dfrac{k_2}{L} \le 10\%, 0 \le \dfrac{k_3}{L} \le 10\% \\ 0 \le \dfrac{y}{L} \le 60\% \\ \sum_{i=1}^{3}\sum_{j=1}^{3} k_i a_{ij} \le L \\ \rho_1 = 0.6611 \ln r_1 + 2.1998, \rho_2 = 0.6603 \ln r_2 + 2.1923, \rho_3 = 0.657 \ln r_3 + 2.1766 \end{cases}$$

2.1.3 Solution of Model

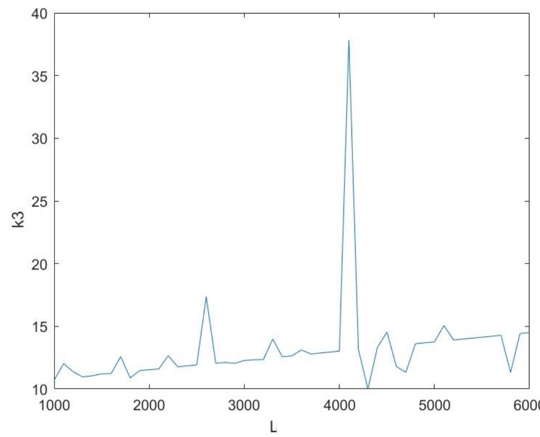
We use the Matlab program to solve the problem, choose L ∈ [1000, 6000] ten thousand yuan as an example, simulate a value of L every 100 intervals, and the running results are shown in the figure:



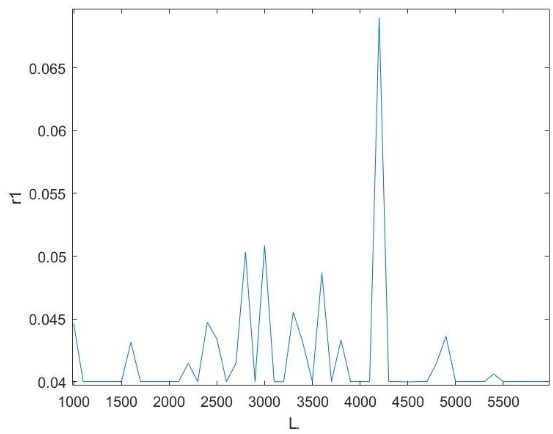
(a) k_1 - L



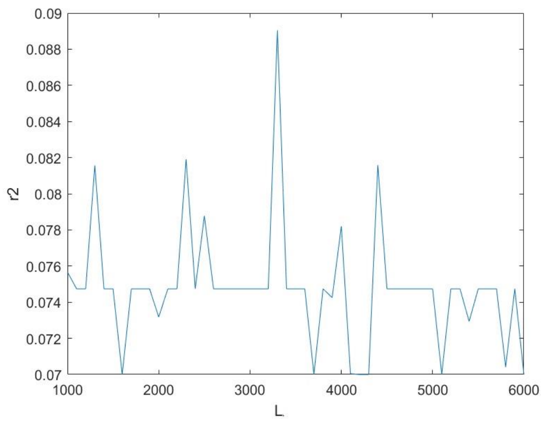
(b) k_2 - L



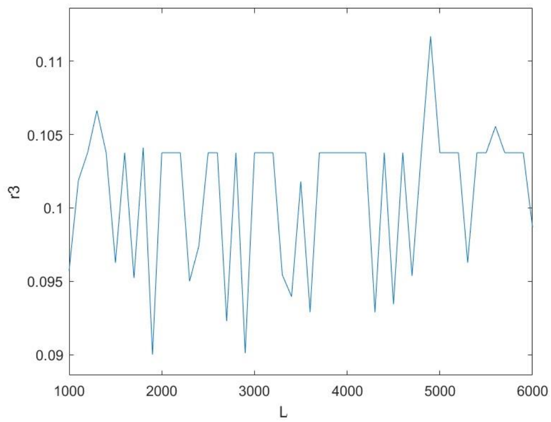
(c) k_3 - L



(d) r1- L



(e) r_2 - L



(f) r_3 - L

Figure 3. Variation of quota k and interest rate r with annual total credit L

Analysis of various loan quota strategies: It can be seen from the figure that there are multiple local optimal solutions for the three in this interval. In this interval, k_3 obtains the global optimal solution at L=41 million yuan. There is no global maximum value for k_1, k_3 in this interval, and it shows a linear increasing trend as L increases.

3. Conclusions

When L changes between 10 and 60 million yuan, the range of k_1 loan amount is [10,75] million yuan, the range of k_2 loan amount is [10,55] million yuan, and the range of k_3 loan amount is [10,40] million Yuan, in line with the actual situation. The total amount of different bank loans should be valued for various types of companies within the corresponding range. When the credit is fixed at a certain value, the optimal allocation quota decision can be obtained.

Analysis of various interest rate strategies: As can be seen from the figure, when L $\in$ [1000, 60] million, the interest rate for normal customers is basically in the range of 4% to 7%; the interest rate for subordinate customers is basically in the range of 7% to 9%, the interest rate for suspicious customers is basically in the range of 9% to 12%. The interest rate values of the three types of customers are quite different. The interest rate of suspicious customers is the most with little fluctuation, the interest rate of sub-class customers is the second and the fluctuation is normal, and the interest rate of normal customers is the least and the fluctuation is large.

When L takes more possible intervals, Matlab images can be used to find the corresponding value range, and it is enough to consider within the range, here only one case is used for explanation.

References

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