

Research on Markowitz Portfolio Model Based on Simultaneous Changes of Means and Variances

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Abstract

This paper mainly studies the Markowitz portfolio model with simultaneous changes in means and variances, and analyzes the influence of simultaneous changes in means and variances on the effective frontier curve. Firstly, this paper introduces the traditional Markowitz portfolio model, which holds that the means and variances of historical stock returns can be used to estimate the effective frontier curve in the future. However, in the real market, the means and variances of stock returns are constantly changing, and there will be deviations when the traditional Markowitz portfolio model is used for analysis. Therefore, this paper establishes a Markowitz portfolio model with simultaneous changes in means and variances, and obtains a new effective frontier curve. At last, the paper downloads the trading data of four stocks in Shanghai Stock Exchange from the financial data terminal of the Hithink RoyalFlush Information Network Co., Ltd, and makes an empirical analysis on the established model.

Keywords

Portfolio Model; Mean-variance Model; Effective Frontier Curve; Stock Yield; Markowitz.

1. Introduction

In recent years, China's economy is in good shape, the securities market is developing at a high speed, and the per capita income is greatly increased. More and more investors are starting to buy financial products such as stocks and funds. With the increase of investment knowledge and the progress of investment concept, investors begin to realize that they should not only pay attention to the benefits but also the risks when purchasing financial products, and learn to weigh the benefits and risks to maximize the investment benefits, which requires investors to have certain knowledge of portfolio management. Portfolio theory was first proposed by Markowitz, an American economist. Markowitz assumed that the means and variances of historical stock returns could be used instead of the means and variances of future returns. However, in the real market, the means and variances of stock returns are not static, and the traditional Markowitz portfolio theory does not consider this change, so there is a deviation between the portfolio derived from the traditional portfolio theory and the realistic optimal portfolio. Based on the practical significance, this paper improves the traditional model, and establishes a portfolio model with means and variances changing at the same time, which makes the portfolio theory more suitable for the real market.

2. literature review

In 1952, Markowitz put forward the mean-variance model. Since then, many scholars have studied the mean-variance model. C.L. He (2011) built a robust static portfolio model based on the uncertainty of model parameters; S. Zhou (2012) found that under certain restrictions, the effective frontier curve of risk-free assets is consistent with that of mean-variance model. Q. Zhang (2013) introduced the constraints in the transaction into the mean-variance model and created the mean-variance model of risk preference coefficient; H.X Yao (2013), considering inflation and other factors, deduced the expression of the effective boundary of the mean-variance model and constructed the investment strategy. Lam, Jaman and Isa(2013) relaxed the hypothesis of normal distribution in the mean-variance model, and introduced the peak and skewness of probability distribution into the mean-variance model.

3. Traditional Markowitz portfolio model

The traditional Markowitz portfolio model has many assumptions: it is assumed that investors are rational, have the same information and understand all the properties of financial products; Assuming that the yield of financial products obeys normal distribution, the yield of each product is related; Assume that the financial market is completely competitive, and so on. According to the above assumptions, Markowitz established a portfolio model based on means and variances. Because this paper does not study risk-free assets, it is assumed that there are N kinds of securities in the financial market, and R is used to express the means of the returns of each kind of securities; v represents the variances of the returns of each kind of securities, and V represents the covariances of the returns of N kinds of securities, that is, the risk brought by buying a unit of securities; W represents the weight of each security in the portfolio; WTR is used to express the returns of portfolio; $1/2WTVW$ is used to represent the risk of portfolio. The traditional Markowitz portfolio model is to minimize portfolio risk under the condition of fixed income. The model is as follows:

$$\min 1/2WTVW$$

$$\text{Constraints are: } W^T R = E_p, \sum_{i=1}^n w_i = 1$$

Where E_p indicates the expected rate of returns.

By solving the model, we can get the corresponding minimum risk with a fixed returns. By drawing the returns and the lowest risk under the returns on the coordinate axis, the effective frontier curve can be obtained, and the points on the curve meet the minimum risk under the condition of fixed returns; The returns is the highest when the risk is fixed. According to the above model, the expression of effective frontier curve is deduced as follows:

$$\sigma_p^2 = \frac{C}{D} \left(E_p - \frac{A}{C} \right)^2 + \frac{1}{C}$$

$$A = e^T V^{-1} R, B = R^T V^{-1} R, C = e^T V^{-1} e, D = BC - A^2$$

Where σ_p^2 indicates the risk of the portfolio; E_p indicates the returns of the portfolio, e indicates N*1 matrix with all data of 1. It can be seen from this expression that with the increase of risk, the corresponding income also increases.

4. Markowitz portfolio model with simultaneous changes in means and variances

Markowitz portfolio model assumes that the means and variances of the returns of each financial product are fixed. However, in the real market, the means and variances of the returns of financial products will change at the same time, and the effective frontier curve derived from the traditional Markowitz portfolio model is no longer applicable. Therefore, it is necessary to establish a Markowitz portfolio model with the means and variances changing simultaneously to provide investors with more accurate investment suggestions. Assume that the means of financial products increase by ΔR and become $R+\Delta R$; The variances of financial product increase by ΔV and become $V+\Delta V$. The established model is as follows:

$$\min 1/2W^T(V + \Delta V)W$$

$$\text{Constraints are: } W^T(R + \Delta R) = E_p, \sum_{i=1}^n w_i = 1$$

The effective frontier curve can be obtained by solving the model, and its expression is as follows:

$$\sigma_p^2 = \frac{C_1}{D_1} \left(E_p - \frac{A_1}{C_1} \right)^2 + \frac{1}{C_1}$$

$$A_1 = e^T (V + \Delta V)^{-1} (R + \Delta R), B_1 = (R + \Delta R)^T (V + \Delta V)^{-1} (R + \Delta R)$$

$$C_1 = e^T (V + \Delta V)^{-1} e, D_1 = B_1 C_1 - A_1^2$$

From the above effective frontier curve, it can be concluded that when the means and variances of financial product returns change simultaneously, the end point $(\frac{1}{C_1}, \frac{A_1}{C_1})$ and bending degree $\frac{C_1}{D_1}$ of the effective frontier curve have changed. According to the model, investors can adjust their portfolio according to the actual market situation.

5. Empirical analysis

5.1 Sample selection and data processing

This paper selects four stocks in the Shanghai Stock Exchange 50 index. They are CITIC Securities Company Limited (600030. SH), China United Network Communications Limited (600050. SH), China State Construction Engineering Corporation Limited (601668. SH) and China Tourism Group Duty Free Corporation Limited (601888. SH). The reason why we choose these four stocks is that they belong to different industries, and the correlation between stocks is relatively low, which meets the conditions of relatively dispersed portfolio.

This paper selects the daily closing prices of four stocks from January 1, 2020 to December 31, 2020 as samples, processes these data, and calculates the returns of each stock and the means and variances of returns respectively. The sample data is from the Hithink RoyalFlush Information Network Co., Ltd "ifnd" financial data terminal.

Because the data selected in this paper is the daily closing price of the stock, it is necessary to use the formula to calculate the returns of each stock. Using MATLAB software to process the data, we can get the means, variances and covariances of the stock returns. According to the means and covariances matrix of stock returns, combined with the effective frontier curve with constant means and variances, the expression of effective frontier curve with constant means and variances can be obtained $\sigma_p^2 = \frac{4789.9}{92.3535} (E_p - \frac{12.9355}{4789.9})^2 + \frac{1}{4789.9}$. It can be seen from the expression that the effective frontier curve with constant means and variances has endpoint $(\frac{1}{4789.9}, \frac{12.9355}{4789.9})$ and bending degree $\frac{4789.9}{92.3535}$. Draw the effective frontier curve with constant means and variances on the coordinate axis, as shown in the following figure:

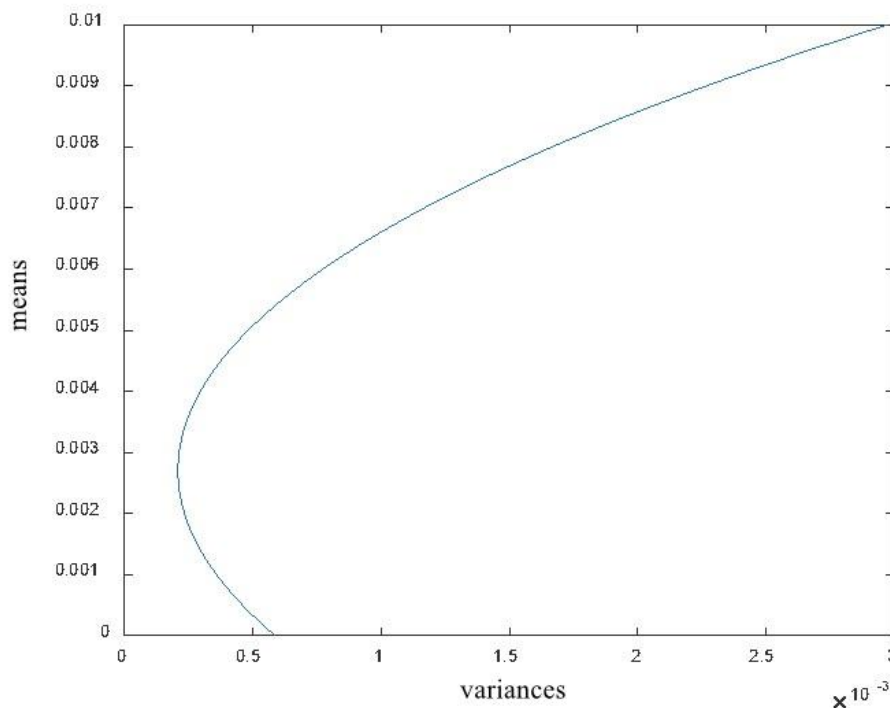


Figure 1. Effective frontier curve with constant means and variances

It can be seen from the graph that investors should choose the point above the endpoint of the effective frontier curve to invest. With the increase of returns, the risk is increasing; the point below the endpoint does not maximize the utility of investors.

5.2 Influence on effective frontier curve

Because the effective frontier curve of the means and variances of the four stocks' returns changes simultaneously is more complex, this paper only studies the effective frontier curve of the means and variances of the returns of CITIC Securities Company Limited. By comparing with the effective frontier curve with the constant means and variances of the returns of CITIC Securities Company Limited, the influence of the means and variances of the returns of CITIC Securities Company Limited on the effective frontier curve can be obtained.

By solving the means and variances of the daily returns of CITIC Securities Company Limited, it can be concluded that the means and variances of daily returns of CITIC Securities Company Limited change at the same time. Assuming that the means of the future returns of CITIC Securities Company Limited increase by 0.001, and the variances change with the change of the means value, the effective frontier curve of the means and variances of CITIC Securities Company Limited can be obtained by introducing the changing data into the Markowitz portfolio model with the means and variances simultaneously changing. The expression of the curve is as follows: $\sigma_P^2 = \frac{4787}{94.0387} (E_P - \frac{14.4}{4787})^2 + \frac{1}{4787}$. From this expression, we can get the effective frontier curve with the means and variances changing simultaneously, and the endpoint is $(\frac{1}{4787}, \frac{14.4}{4787})$, the bending degree is $\frac{4787}{94.0387}$.

In order to more clearly observe the impact of 0.001 change in the means of returns on the effective frontier curve, this paper draws the effective frontier curve in two cases in the same graph, in which the blue represents the Markowitz effective frontier curve with the means and variances unchanged, and the red represents the Markowitz effective frontier curve with the means and variances changing simultaneously. It can be seen from the graph that the effective frontier curve with the means and variances changing simultaneously is more curved, the symmetry axis is more upward, and the minimum variances point is more right. For investors, the uncertainty of returns and risk is greater.

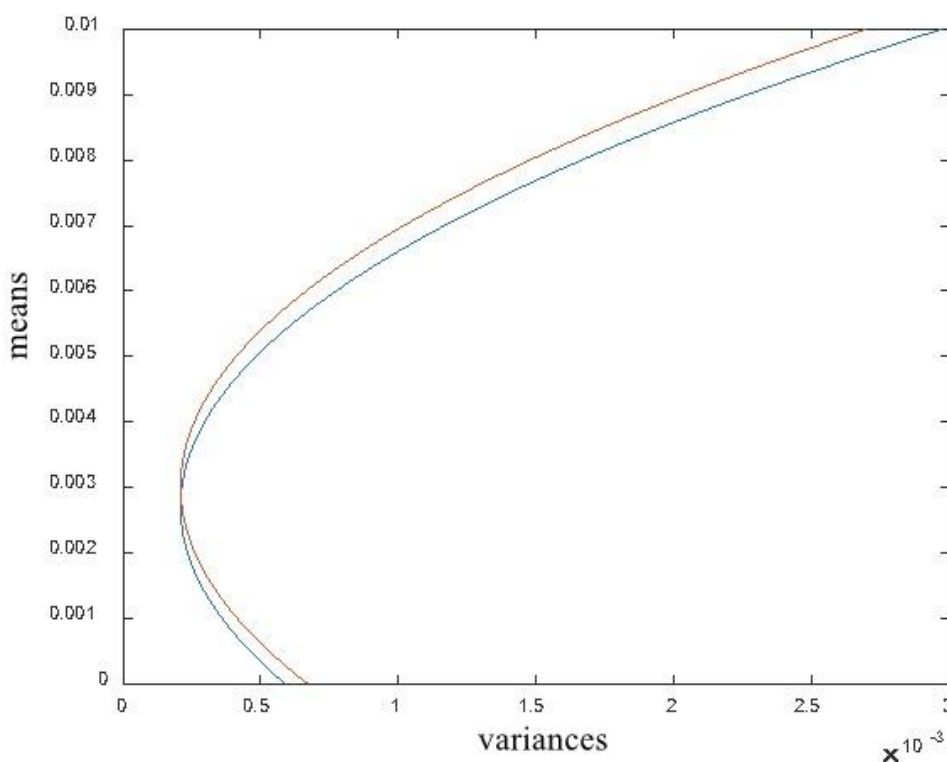


Figure 2. Effective frontier curve of means and variances changing simultaneously

6. Research Summary

The portfolio model proposed by Markowitz has many restrictions. Assuming that the means and variances of the returns on assets remain unchanged, the current asset allocation can be guided by the previous portfolio model. However, in the real market, the means and variances of the returns on assets will change at the same time. Therefore, this paper establishes the Markowitz portfolio model with the means and variances simultaneously changing, and obtains the effective frontier curve. Through the comparison between the effective frontier curve with constant means and variances and the effective frontier curve with simultaneous change of means and variances, it can be concluded that the increase of means and variances will lead to the bending degree of the effective front curve, the symmetrical axis will move upward, and the minimum variances point will move to the right. Therefore, investors in portfolio management must pay attention to the influence of the means and variances of returns on assets on investment utility, so as to reduce the uncertainty of investment.

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