Research on the Search Problem of Optimal Communication Network under Uncertainty

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Abstract

The graph is the research object, based on the general traffic communication network in each path travel time of each section of the mean and variance of known, on the basis of constructing the optimum path reliability model, to ensure that the same high probability shortest total time arrived, and then integrate the time correlation and spatial correlation model, layer upon layer solution. Finally, according to the established model analysis algorithm, the optimal path.

Keywords

Optimal Path; Normal Distribution; Moren Model; Sensitivity Analysis.

1. Introduction

In recent years, the rapid economic growth and the acceleration of urbanization have led to a continuous increase in the urban population. The large concentration of urban population and the surge in the number of private cars per capita have made urban traffic facing more and more difficult tasks, such as traffic congestion, traffic safety, and As soon as high-efficiency transportation and other problems emerged, the transportation problems became increasingly severe, as can be seen from the annual "Spring Festival Transport" phenomenon, which not only has a serious impact on residents' daily life, but also restricts the development of the city and surrounding cities. In a complex traffic environment, how to comprehensively consider various factors to find a reliable, fast, and safe optimal path has become an urgent problem for everyone.

In a specific area, according to the development of regional economy and the needs of people's activities, various modern transportation modes are combined, various transportation lines and traffic points are interwoven, forming different forms and levels of transportation network, referred to as transportation network.Its layout is influenced and restricted by economy, society, technology and nature.

Classified by means of transportation, a railway transportation network, a road transportation network, a waterway transportation network, an air transportation network, and a pipeline transportation network have been formed. Different transportation modes are combined to form a comprehensive transportation network.

The traditional optimal path is the path with the shortest average total travel time based on the analysis of ideal traffic conditions.In this case, the traveling time of each road section is determined, and the classic shortest path algorithm (Dijkstra algorithm) can be used to find the shortest path.This is also the algorithm used by most vehicle path navigation systems to find the optimal path.However, the traditional path has considerable disadvantages. In real life, a series of uncertain factors such as traffic accidents, weather conditions and flow will make the optimal path lose its optimality.

Different from the traditional path, the improved optimal path takes into account a series of uncertain factors that may be encountered in real life, such as weather and traffic flow, etc., and makes the driving time of each road section uncertain and approximately obeys a random distribution. The model is established to give a relative optimal path.

2. Methods

2.1 Preparation of the model

The definition of temporal correlation and spatial correlation is as follows:

2.1.1 Time correlation

For section A, the correlation of different time periods, such as the correlation between 7:00-8:00 and 8:00-9:00.

The following introduces the Copula function to describe the time correlation. Copula can be interpreted as "dependency function" or "connection function", which is a function that connects the joint distribution of multi-dimensional random variables with its one-dimensional marginal distribution. The following first introduces Sklar's theorem.

Sklar's theorem:

Suppose that the marginal distribution function of a multidimensional distribution function is:

$$F_1(\bullet), F_2(\bullet), F_3(\bullet) \cdots F_n(\bullet),$$

Then there exists a Copula function that satisfies:

$$F(x_1, x_2, x_3 \cdots x_n) = C(F_1(\bullet), F_2(\bullet), F_3(\bullet) \cdots F_n(\bullet))$$

If $F_1(\bullet), F_2(\bullet), F_3(\bullet) \cdots F_n(\bullet)$ continuous, the Copula function is uniquely deterministic, and vice versa.According to this theorem, when the marginal distribution of travel time of multiple sections is determined and an appropriate Copula function is selected, the joint distribution of travel time of these sections can be easily calculated, which is the advantage of Copula function to solve the reliability of travel time of traffic network in practice.

2.1.2 Spatial correlation

Spatial correlation is measured by the matching of position similarity and attribute similarity. The similarity of position can be described by spatial proximity matrix or weight matrix, while the similarity of attribute value is generally described by cross product or square difference, or absolute difference. If there is positive spatial autocorrelation, the difference of attribute values in the spatial position of the nearest neighbor is small. If there is a negative spatial autocorrelation, the attribute values of the nearest neighbor vary greatly. In addition, the degree of spatial autocorrelation is different, and its intensity is measurable.Strong spatial autocorrelation means that the attribute values of the nearest neighbor objects are highly close, regardless of whether they are positive or negative. The measurement of spatial autocorrelation is based on the binary logic of adjacency of spatial units.According to this definition, the structure of adjacent edge is expressed by: spatial adjacency refers to the boundary of non-zero length shared by two spatial units, thus giving the spatial closeness degree of 1.Usually, a binary symmetric space weight matrix is defined to express the proximity of space regions of three positions, and its form is as follows:

$$W = \begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & & \vdots \\ w_{n1} & \cdots & w_{nn} \end{bmatrix}$$

Where: w_{ij} is the proximity relationship between region i and j, which can be measured according to the adjacency standard or distance standard.

2.2 Model establishment

As for the uncertainty of road sections, previous studies generally only provide the optimal path solving algorithm considering only general road conditions (the road traffic conditions here do not include the conditions that have a great impact on driving time).Our innovation is that we take into account traffic accidents, bad weather, emergencies, and so on, and we look at both temporal and spatial correlations.And the control variable method is used, that is, when considering the temporal correlation of this section, the influence of spatial correlation on it is not considered.Temporal correlation is not taken into account in the analysis of spatial correlation.

The Moran model is established, and its expression is as follows:

$$I = \frac{n}{\sum\sum W_{ij}} \bullet \frac{\sum\sum W_{ij}(X_i - \bar{X})(X_j - \bar{X})}{\sum(X_i - \bar{X})^2}$$

Where: n is the number of research objects, X_i is the observed value; $\bar{X}$ is the average value of X_i, and $Moran'sI$'s expected value is:

$$E(I) = -\frac{1}{n-1}$$

The variance is:

$$Var(I) = \frac{n^2 S_1 - nS_2 + 3(\sum_{ij}\sum W_{ij})^2}{(\sum_{ij}\sum W_{ij})^2(n^2-1)}$$

Where:

$$S_1 = \frac{1}{2}\sum_{ij}\sum(W_{ij} + W_{ji})^2$$

$$S_2 = \sum_i(\sum_i W_{ij} + \sum_j W_{ji})^2$$

The spatial autocorrelation statistic used in this paper is derived from A, that is, the test statistic Z is obtained by standardizing the calculated A value on the premise of the approximate normal hypothesis.According to the z-value of the null hypothesis "the variables are randomly distributed in the discussed spatial region", the test statistic Z is obtained.Then the null hypothesis is tested according to the z-value to determine whether the spatial autocorrelation exists.Significance level was taken during the test (two-sided test).Among them:

$$Z = [I - E(I)]/[VAR(I)]^{1/2}$$

When the Z value is positive and significant, it indicates that there is a positive spatial autocorrelation, that is, similar observations (high or low values) tend to cluster in space; when the Z value is negative and significant, it indicates that there is a negative space Autocorrelation, similar observations tend to be distributed; when the Z value is zero, the observations are distributed independently and randomly. An example is given below to illustrate:

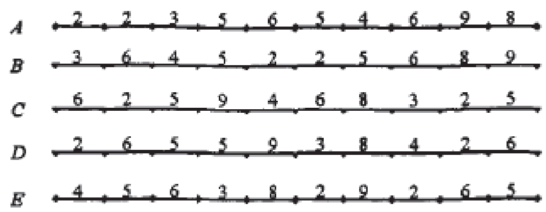

Figure 1. Traffic accident map of each road section

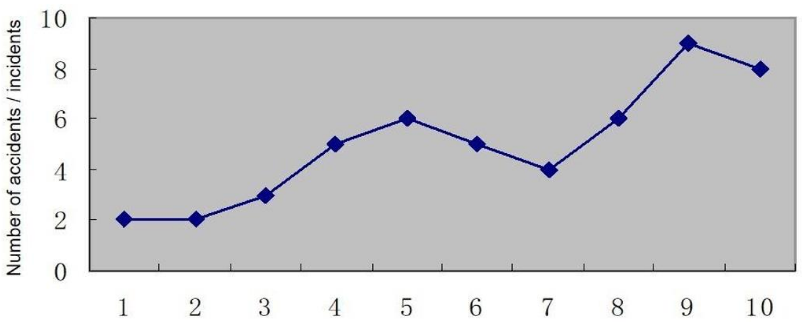

Figure 2. Line diagram of traffic accidents in section A

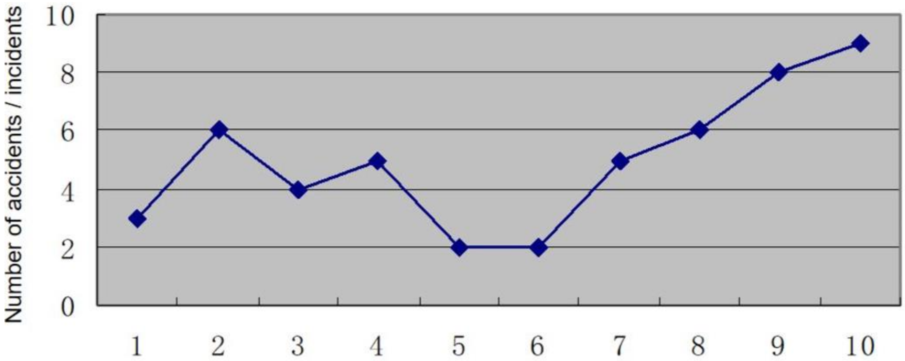

Figure 3. Line diagram of traffic accidents in section B

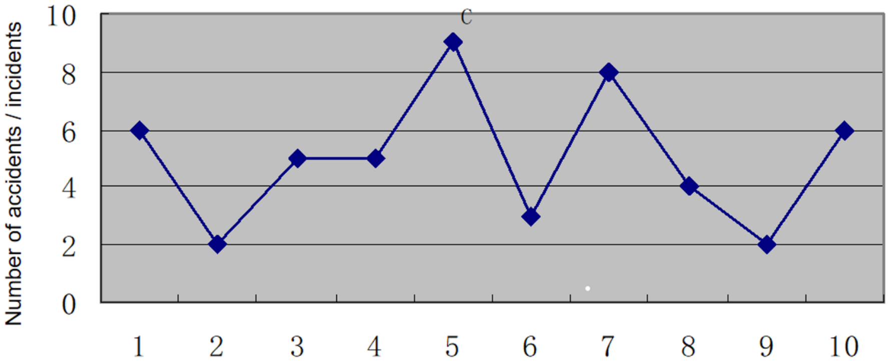

Figure 4. Line diagram of traffic accidents in section C

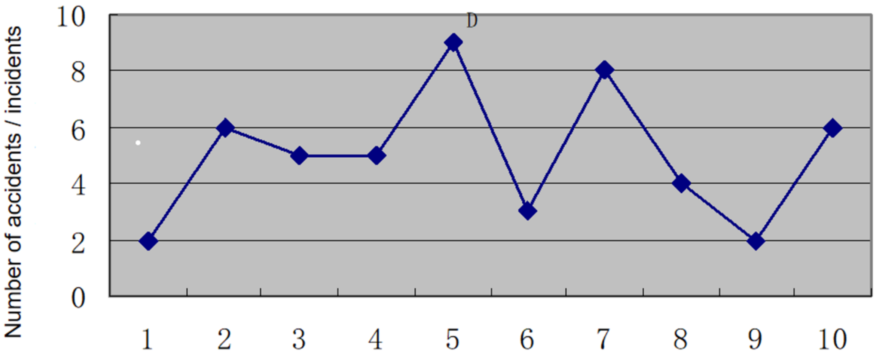

Figure 5. Line diagram of traffic accidents in section D

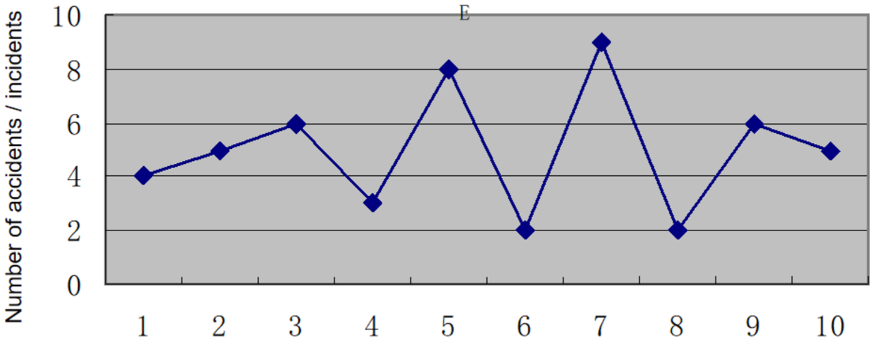

Figure 6. Line diagram of traffic accidents in section E

The accident spatial autocorrelation index on these 5 road sections is obtained by calculation, and the results are as follows:

Table 1. Road section autocorrelation index

Road section	Moran's I	Z
A	0.667	2.63
B	0.467	1.95
C	-0.111	0.00
D	-0.444	-1.13
E	-0.978	-2.93

As can be seen from the above table, at the significance level of 0.01 ($Z = 2.58$), A is positive autocorrelation, and E is negative autocorrelation; at the significance level of 0.05 ($Z = 1.96$), A and B are positive autocorrelation, and D, E are negative autocorrelation. Correlation; C is non-autocorrelation.

3. Sensitivity analysis

For the analysis under a certain variance with different probabilities: if the model is guaranteed from the fast route around the city, the time required is $t_1 = \mu_1 + u_p\sigma_1$.

The time required from the urban road is $t_2 = \mu_2 + u_p\sigma_2$.

Table 2. Quantiles of the normal distribution

data	1	2	3	4	5	6	7	8	9	10
Probability	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
digits	0	0.13	0.26	0.39	0.53	0.68	0.85	1.04	1.29	1.65

It can be seen that the probability increases with the increase of quantile. Matlab is used to make relevant images:

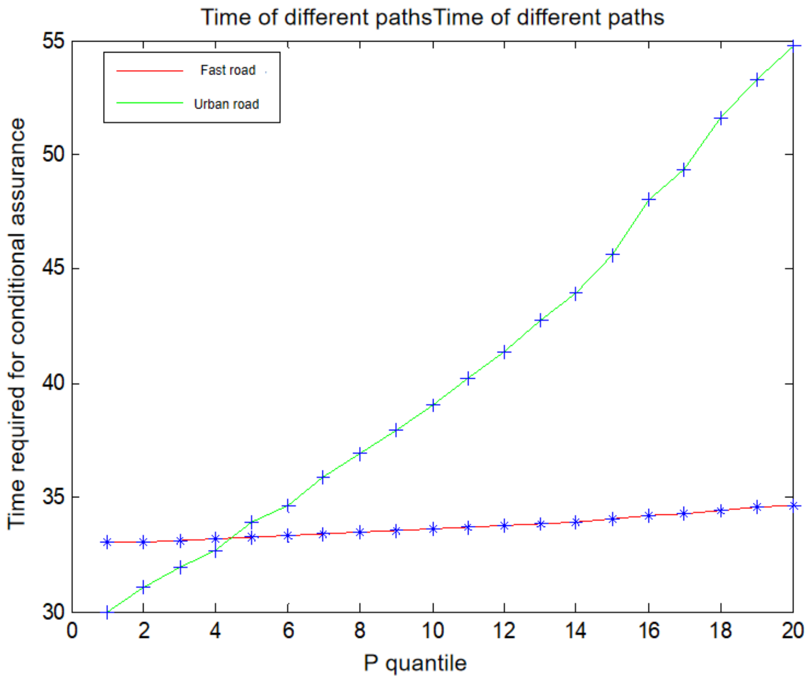

Figure 7. Time chart of conditional guarantee

As can be seen from the above routes, the time required for the two roads both tends to increase when the probability of reaching the destination increases. However, the urban road has an obvious increase trend with the increase of probability. There is too much randomness and more time is delayed on the way. It is interesting to find that when the probability is 0.57, the arrival time from the two ways is the same. But when the probability of reaching the destination is lower than 0.57, the urban road should be selected to gain the chance of road congestion free. However, when the probability is high, the express road should be selected to reduce the interference of random factors on the way.

As a result, it is possible to reach the road sections where the random factors of multiple road sections obey the normal distribution, and use the convolution formula to synthesize a path, which also satisfies the above-mentioned sensitivity analysis.

4. Conclusion

Positive autocorrelation has similar accident probability in adjacent sections of highway.It can be shown that certain factors on this road segment (such as weather or design factors, etc.) have a causal relationship with the increase in accidents on adjacent roads. Negative network autocorrelation can indicate the trend of different accident rates in adjacent sections of highways. This is the result of analysis, but rarely occurs in observational tests, unless there is an exit or access at all other sections of the road that would cause additional accidents on those sections.Therefore, after investigating the number of accidents in known sections, the spatial correlation of outbound traffic roads can be obtained.

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