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Study on Dynamic Characteristics of Stick-slip and Bit-bounce in Drill-string Systems

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Abstract

To study the dynamical properties and control methods of drill-string systems, it is crucial to address the problems of fatigue failure of the drill-string and low drilling efficiency caused by stick-slip and bit-bounce in drill-bits. Firstly, an electro-mechanical coupled dynamics model for the drill-string system is developed, which takes into account the coupling relationship between the asynchronous motor, drill-string, drill-bit, and rock. Secondly, we develop a dynamic control approach for the drill-string system by using the sliding mode control principle and the hook-load control function. The effects of weight on the bit, rotation speed, and rock stiffness on the drill-bit motion are then combined with the motion relations of the rotary table, drill-string, and drill-bit. The relationship between the properties of the motion of the drill-bit and the dynamics of the drill-string system is demonstrated using an electro-mechanical coupled dynamical model of the drill-string system. Finally, the motor speed and hook load are controlled to address stick-slip, bit-bounce, and combination of stick-slip and bit-bounce motions. The results show that parameters such as rotation table speed, drill-bit type, rock stiffness and drilling depth significantly affect the dynamics of the drill-string system. The stick-slip and bit-bounce motions, which are more detrimental to the safety of the drill-string structure, can induce sharp fluctuations in the torque of the cutting resistance, the dynamic weight on bit and the axial displacement.

Keywords

Drill-string; Stick-slip; Bit-bounce; Dynamics; Control.

1. Introduction

The study of drill-string dynamics is of great significance in the design and control of borehole trajectory, the checking of drill-string strength, the optimization of the drill-string structure, and drilling parameters [1]. The stick-slip and bit-bounce motions of the drill-string system are the direct factors leading to the fatigue failure of the drill-string and the reduction of drilling efficiency [2]. Therefore, the research on the dynamic characteristics of the drill-string system and control is an important measure to reduce the risk of drill-string failure and improve drilling efficiency[3, 4]. Jansen reduced the dynamical model of the drill-string to a non-equilibrium rotor system by neglecting the interaction between the drill-bit and the rock, since he found that the rotation of the drill-string could cause violent vibrations in the drill-collar; Spanos [6], Batako [7], Zhu [8], Zhang [9] and Di [10] et al. proposed several classical friction models to describe the coupling between the drill-bit and formation and established a dynamic model of bottom hole assembly. However, this model neglects the effect of the motion of the upper assembly on the mechanical behavior of the lower assembly; Fu Meng [11] et al. established the dynamic model of the drill-string in the whole section and explored the energy variation law of the stick-slip vibration system of the drill-string without considering the coupling effect of fluid media flow and drill-string vibration. Christoforou [12] established the dynamic system model of the drill-string system driven by a DC motor in the whole section and studied the relationship between parameters such as rotary table speed, weight on bit and rate of penetration, and drill-bit motion. To control the motion characteristics of the drill-string system, Ritto [13-15] proposed the PID method to control the output torque of the rotary table to solve the problem of the stick-slip motion of the bottom hole assembly. However, it is impractical to tune the output torque of the rotary table to control the stick-slip motion of the bottom hole, since the driving torque of the drill-string system is provided by the motor. Kamel and Yigit [16] established the control model of the whole drilling-string system driven by a DC motor and solved the problems of stick-slip and bit-bounce by controlling the motor torque. In summary, while some progress has been made in the previous studies on the dynamics and control of drill-string systems, the research on the dynamics and control of drill-string systems driven by asynchronous AC motors has yet to be elucidated.

Thus, a dynamical model for the electromechanical coupling of the drill-string system is developed by considering the coupling relations between the asynchronous motor, drill-string, drill-bit and rock; combining the motion relations of the rotary table, drill-string, and drill-bit, the effects of weight on the drill-bit motion, rotational velocity, and rock stiffness have been studied; the relationship between the kinematic properties of the drill-bit and the dynamical properties of the drill-string system has been explored using an electro-mechanical coupled dynamical model of the drill-string system; Finally, the problem of stick-slip, bit-bounce, and combination of stick-slip and bit-bounce motions is solved by controlling the motor speed and the hook load.

2. Dynamic Model of the Drill-string System

The dynamic model of the drill-string system is mainly composed of the lifting system, AC asynchronous motor, reducer, drill-pipe, stabilizer, drill-collar, drill-bit, and formation as shown in Fig. 1. The lifting system is used to control the dynamic weight on bit of the drill-string and the asynchronous motor drives the rotation motion of the drill-string.

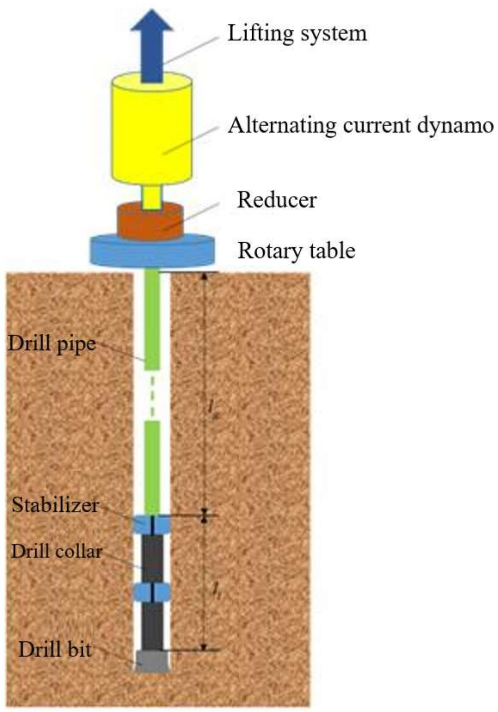

Fig. 1 Dynamic model of the drill-string system

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The flux equation of asynchronous motor in rotor synchronous coordinate system (d , q) is:

$$\begin{aligned}\psi_{ds} &= L_s i_{ds} + L_m i_{dr} \\ \psi_{qs} &= L_s i_{qs} + L_m i_{qr} \\ \psi_{dr} &= L_m i_{ds} + L_r i_{dr} \\ \psi_{qr} &= L_m i_{qs} + L_r i_{qr}\end{aligned} \tag{1}$$

where, ψ_{ds} and ψ_{qs} are the magnetic flux of the stator d axis and q axis; ψ_{dr} and ψ_{qr} are the magnetic flux of D axis and Q axis of the rotor; i_{ds} and i_{qs} are the currents on the d and q axes of the stator, respectively; i_{dr} and i_{qr} are the currents on the d and q axes of the rotor, respectively; L_s and L_r are the inductances of stator and rotor, respectively; L_m is the mutual inductance of stator and rotor. At the static operating point, the voltage balance equation of the induction motor can be expressed as:

$$\begin{aligned}u_{ds} &= R_s i_{ds} - \omega_e \psi_{qs} \\ u_{qs} &= R_s i_{qs} - \omega_e \psi_{ds} \\ 0 &= R_r i_{dr} \\ 0 &= R_r i_{qr} + \left(\omega_e - \omega\right)\psi_{dr}\end{aligned} \tag{2}$$

where, R_s and R_r are the resistance of stator and rotor, respectively; ω_e is the power supply frequency of the grid; ω is the angular velocity of the motor; u_{ds} and u_{qs} are d and q axis voltages, respectively. The output torque of AC asynchronous motor according to the motor flux equation and the theory of electrical machinery is given by:

$$\begin{aligned}T_m &= K_m i_{qs} \\ K_m &= (3/2) n_p (L_m / L_r) \psi_{dr}\end{aligned} \tag{3}$$

where, the torque and n_p are pole-pairs. According to the mechanical model of the drill-string system, the differential equation of the coupling motion of the rotary table, reducer, and motor can be expressed as:

$$\left(J_r + n^2 J_m\right)\ddot{\varphi}_r + c_r \dot{\varphi}_r + k_\varphi\left(\varphi_r - \varphi\right) = nT_m \tag{4}$$

$$\dot{\varphi}_r = \omega / n \tag{5}$$

where, J_r is the moment of inertia of the rotary table; $k_\varphi = G\pi\left(d_{gw}^4 - d_{gn}^4\right)/\left(32 l_g\right)$ is the torsional stiffness of the drill-string; φ and φ_r are the rotational angular displacement of the drill bit and the turntable, respectively; n is the transmission ratio of the gearbox; c_r the torsional damping coefficient of the drill-string; $(\dot{\ }) = d()/dt, (\ddot{\ }) = d^2()/dt^2$.

Due to the anisotropy of the formation rock properties and the cutting process of the drill-bit, the torque required by the drill-bit to cut rock has obvious nonlinear characteristics in the process of rotating to break rock. Therefore, torque on bit is a function of the bit axial dynamic weight on bit F_r and cutting parameters [16].

$$T_b = F_r r_b \left(\mu(\dot{\varphi}) + \xi \sqrt{\frac{\delta_c}{r_b}} \right) \tag{6}$$

where, r_b is the radius of the drill-bit; δ_c is the depth of cut per revolution of the drill-bit; μ and ξ are the friction coefficient and cutting behavior coefficient of the drill-bit cutting rock, respectively; The experimental results show that the friction behavior between the drill-bit and formation is a nonlinear function of the rotational speed of the drill-bit $\dot{\varphi}$, which can be expressed as [3].

$$\mu(\dot{\varphi}) = \mu_0 \left(\tanh \dot{\varphi} + \frac{\alpha \dot{\varphi}}{\left(1 + \beta \dot{\varphi}^{2\gamma}\right)} + \nu \dot{\varphi} \right) \tag{7}$$

where, μ_0, α, β, γ and ν are parameters of the friction model determined by the friction test between the drill-bit and formation. δ_c, which represents the depth of cut per revolution of the drill-bit determined by the average rate of penetration ω_p and average rotate velocity ω_a, can be expressed as:

$$\delta_c = \frac{2\pi\omega_p}{\omega_a} \tag{8}$$

It can be seen from the above equation that the larger the static weight on bit and average drill-bit speed are, the faster the rate of penetration of the drill-bit is. The average rate of penetration can be expressed by static weight on bit F_0, average bit speed ω_a, and rock mechanical properties c_1 [3].

$$\omega_p = c_1 F_0 \sqrt{\omega_a} + c_2 \tag{9}$$

where, c_1 and c_2 are the friction process and cutting behavior of the drill-bit when cutting rock, depending on the type of drill-bit and rock mechanical properties. The differential equation of the coupling motion of drill-pipe, drill-collar, drill-bit, and rock can be expressed as:

$$J\ddot{\varphi} + c_\varphi \dot{\varphi} + k_\varphi \left(\varphi - \varphi_r \right) = -T_b \tag{10}$$

where, J is the sum of the moment of inertia of drill-collar and drill-pipe and can be expressed as:

$$J=\frac{\rho\pi\left(d_{tw}^{4}-d_{tn}^{4}\right)l_{t}}{32}+\frac{1}{3}\frac{\rho\pi\left(d_{gw}^{4}-d_{gn}^{4}\right)l_{g}}{32} \tag{11}$$

where, G is the shear modulus of the material.

In the axial direction of the drill-string, the hook block supports most of the weight of the drill-pipe and drill-collar, and a small part of the weight of the drill-collar acts on the drill-bit as weight on bit F_0. In the static case, the weight on bit is equal to the difference between the total weight of the drill-string and the load of the hook block, and can be expressed as:

$$F_{0}=m_{z}g-F_{h} \tag{12}$$

where, F_h is the hook load. m_z is the axial effective mass of the drill-string, which can be expressed as:

$$m_{z}=\frac{\rho\pi\left(d_{tw}^{2}-d_{tn}^{2}\right)l_{t}}{4}+\frac{\rho_{fl}\pi\left(d_{tn}^{2}+C_{a}d_{tw}^{2}\right)l_{t}}{4}+\frac{\rho\pi\left(d_{gw}^{2}-d_{gn}^{2}\right)l_{g}}{12} \tag{13}$$

where, ρ is the material density of drill-collar and drill-pipe; ρ_{fl} is the drilling fluid density; d_{tw} and d_{tn} are the external and inner diameters of the drill-collar, respectively; l_t and l_g are the length of drill-collar and drill-pipe, respectively; C_a is the additional mass coefficient of drill-collar. During the drilling phase, the drill-bit bears not only weight on bit .., but also the rock reaction force on the drill-bit F_r. The axial load of drill-string system F_z can be expressed as:

$$F_{z}=F_{0}-F_{r} \tag{14}$$

where, the reaction force F_r is related to the rock properties and the axial displacement of the drill-string and can be expressed as:

$$F_{r}=\begin{cases}k_{c}\left(z-\chi\right), & if\ z\geq\chi\\ 0, & if\ z<\chi\end{cases} \tag{15}$$

where, k_c is the contact stiffness coefficient of formation rock; z is the axial displacement of drill-string; χ is the bottom hole mode of rock cutting by the drill-bit and can be expressed as:

$$\chi=\chi_{0}f\left(\varphi\right) \tag{16}$$

where, $f\left(\varphi\right)$ is the bottom hole mode function depending on the type of the drill-bit to simulate the cut marks of rock cutting by the drill-bit. Previous studies have shown that the cut mark ripple on the rock surface is a periodic function of the bit rotation phase. Assuming that $f\left(\varphi\right)$ is a sine function related to φ [3].

$$f(\varphi)=\sin(b\varphi) \tag{17}$$

where, b = 1 is used for PDC bits and b = 3 is used to represent tri-cone drill-bits. According to Newton's second law, the differential equation of the axial vibration of the drill-string system can be expressed as:

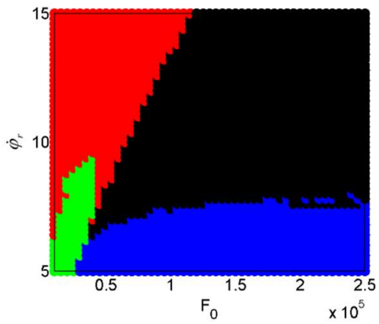
$$m_z\ddot{z}+c_z\dot{z}+k_z z=-F_z \tag{18}$$

where, $k_z=E\pi\left(d_{gw}^2-d_{gn}^2\right)/\left(4l_g\right)$ is the axial stiffness coefficient; c_z is the axial damping coefficient of the drill-string; E is the elastic modulus of the material.

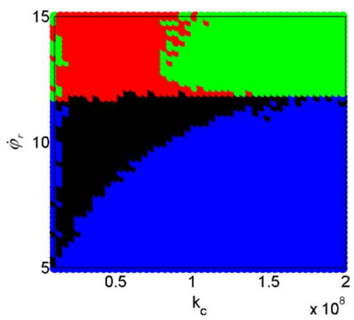
3. Research on Motion Characteristics of the Drill-bit

The motion model of the drill-string system is established according to Eqs. (4), (10) and (18) to study the influence of parameters such as weight on bit, drill-pipe length, drill-collar length and rock stiffness on the motion of PDC bit and cone drill-bit. The parameter values used in the calculation of the model are $J_r=923$, $J_m=23$, $c_{rt}=0$, $c_a=30000$, $c_v=3.75$, $n=7.2$, $\chi_0=0.01$, $c_1=1.35\times10^{-8}$, $c_2=-1.9\times10^{-4}$, $\alpha=2$, β=1, $v=0.01$, $m_0=0.06$, $\xi=1$, $r_b=0.22$, $g=9.8$, $E=210\times10^9$, $G=78\times10^9$, $\rho=7850$, $d_{tw}=0.2286$, $d_{tn}=0.0726$, $\rho_{fl}=1450$, $c_{am}=1.7$, $d_{gw}=0.127$, $d_{gn}=0.095$.

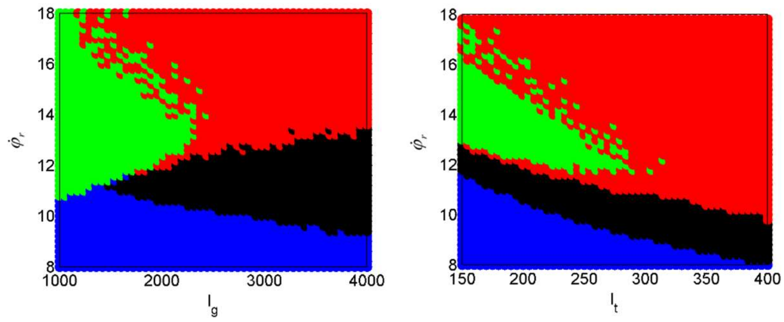
The relationship between the PDC bit motion and parameters such as rock stiffness, rotary speed, weight on bit, drill-pipe and drill-collar length is shown in Fig. 2. The red area represents the bit-bounce motion; the green area represents normal drilling motion; the black area represents the stick-slip motion of the drill-bit; the blue area represents the stick-slip motion of the drill-bit (the color properties are the same as in Fig. 3 below). Fig. 2(a) shows the influence of rotary speed and weight on bit variation on the PDC bit motion. When the PDC bit encounters hard formation in the shallow well, reducing rotation speed and weight on bit is conducive to normal drilling. Fig. 2(c) represents the influence of rotary speed and drill-pipe length (well depth) changes on PDC bit motion. The PDC bit cannot be drilled properly in a deep well with soft formations. Fig. 2(d) the influence of rotary table speed and drill-collar length variation on the PDC bit motion. When the PDC bit encounters soft formation in the shallow well, it is not suitable to use the bottom-hole assembly that is too long. In summary, reducing the rotation speed, weight on the bit, and length of the bottom hole assembly facilitates normal drilling of the drill-bit when the PDC bit encounters soft formations in shallow wells, but makes drilling difficult when the PDC bit encounters soft formations in deeper wells.



(a) $k_c=5\times10^7, l_g=2000, l_t=200$



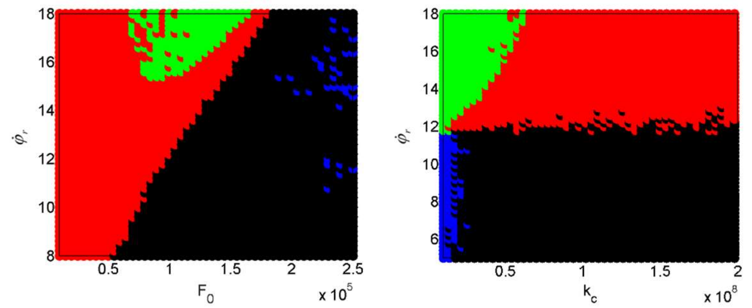
(b) $F_0=1\times10^5, l_g=2000, l_t=200$



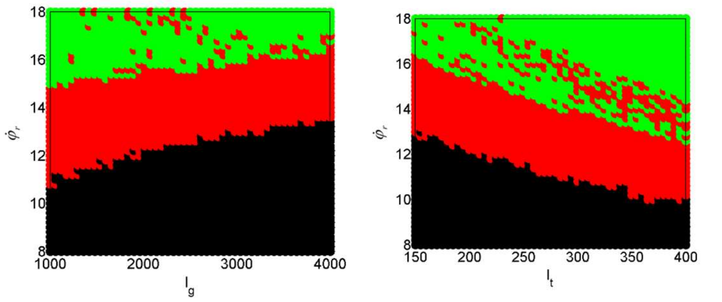
(c) $F_0 = 1\times10^5, k_c = 1\times10^8, l_t = 200$ (d) $F_0 = 1\times10^5, l_g = 2000, k_c = 1\times10^8$

Fig. 2 Motion characteristics of the PDC bit

The relationship between rock stiffness, rotary speed, weight on bit, drill-pipe and drill-collar length, and cone bit motion is shown in Fig. 3. Fig. 3(a) shows the influence of rotary table speed and weight on bit variation on the cone bit motion. When the cone bit encounters soft formations in the shallow well, increasing the rotation table speed and weight on bit ensures normal drilling of the dill-bit. The influence of rotating speed and rock stiffness on the motion of the cone bit is shown in Fig. 3(b). By increasing rotation speed and increasing weight on bit, it is beneficial for the cone bit to drill normally when the cone bit encounters hard formations in the shallow well. Fig. 3(c) shows the influence of the rotation speed of the rotary table and the length of the drill-pipe on the motion of the cone bit. Increasing the rotary speed is beneficial to the normal drilling of the cone bit. The influence of rotary table speed and drill-collar length on cone bit motion is shown in Fig. 3(d). Increasing the length of the bottom hole assembly and the rotation speed favors normal drilling of the cone bit when it encounters soft formations in shallow wells. In summary, increasing the rotation speed, the weight on the bit, and the length of the bottom hole assembly favors normal drilling of the bit when the cone bit encounters soft formations in shallow wells; Heavy weight on bit is detrimental to the normal drilling of the cone bit when encountered in hard formations in a shallow well.



(a) $k_c = 5\times10^7, l_g = 2000, l_t = 200$ (b) $F_0 = 1\times10^5, l_g = 2000, l_t = 200$



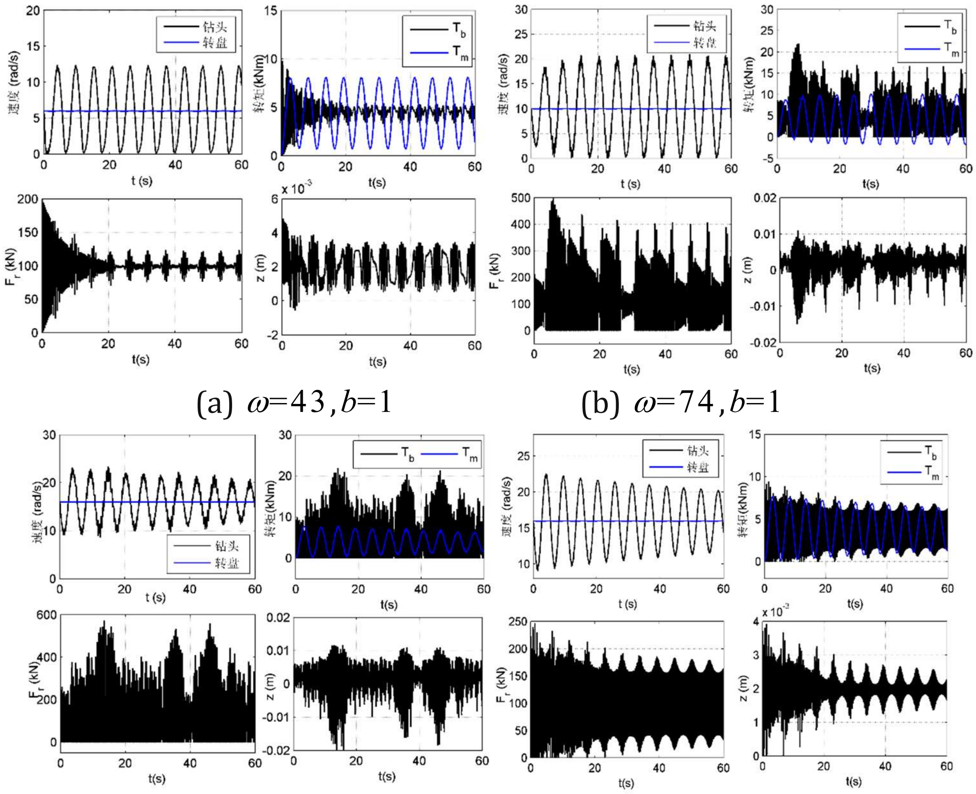
(c) $F_0 = 1\times10^5, k_c = 5\times10^7, l_t = 200$ (d) $F_0 = 1\times10^5, l_g = 2000, k_c = 5\times10^7$

Fig. 3 Motion characteristics of the cone bit

4. Research on Dynamic Characteristics of the Drill-string

The electro-mechanical coupling dynamic model of the drill-string is established as deduced by Eqs. (2), (4), (10) and (18) to analyze the dynamic characteristics of the drill-string. The weight on bit is 1×10^5 N; the length of the drill-pipe is 2000 m; the length of the drill-collar is 200 m; the stiffness of the rock is 5×10^7 N/m. The power rating, nominal voltage, rated speed, and resistance of the rotor and stator of the AC induction motor are 400 HP, 600 V, 120 rad/s, $0.58\ \Omega$ and $0.9\ \Omega$, respectively. The grid supplies power at a frequency of 50 Hz; The rotor and stator have a self-inductance of 0.6H and a mutual inductance of 0.3H; the pole number of the motor is 3.

When the motor speed is 43 rad/s and the speed of the rotary table is reduced to 6 rad/s through the reducer, the dynamic characteristics of the drill-string system are shown in Fig. 4(a); The velocity fluctuations of the PDC sites range from 0-13 rad/s in the stable phase of the system; when the rotational speed of the drill-bit is equal to zero, the drill-bit and rock remain in a viscous state, and when the motor output torque T_m is greater than the cutting resistance torque T_b of the drill-bit, the drill-bit and rock slide relatively; The process by which the bit and rock go from sticking to sliding is known as stick-slip motion, which is prone to fatigue failure of the drill-string and drill-bit; when the drill-bit is in a viscous state, the dynamic weight on bit F_r is approximately equal to the static weight on bit F_0, while when the drill-bit is in the sliding state, the dynamic weight on bit F_r fluctuates continuously; it can be seen that the stick-slip motion of the drill-bit inevitably leads to fluctuations of the weight on bit. When the motor speed is 74 rad/s and the speed of the rotary table is 10 rad/s, the dynamic characteristics of the drill-string system are shown in Fig. 4(b); during the stability phase of the system, the velocity fluctuation of the PDC bit ranges from 0 to 20 rad/s, so the dril-bit is in stick-slip motion; when the cutting resistance torque T_b of the drill-bit is zero, the drill-bit is out of contact with the rock, causing the bit-bounce; It can be seen that there are stick-slip and bit-bounce phenomena when the drill-bit drills into the formation, and the motion of the drill-string system can be called stick-slip and bit-bounce motion; compared with the stick-slip motion of the drill-string system in Fig. 4(a), the fluctuations of the cutting resistance torque, dynamic weight on bit and axial displacement induced by stick-slip and bit-bounce motions of the drill-string are more drastic, so the stick-slip and the bit-bounce motions are more harmful to the safety of the drill-string structure. When the motor speed continues to increase to 118 rad/s and the speed of the rotary table is 16 rad/s, the dynamic characteristics of the drill-string system are shown in Fig. 4(c); In the stable phase, the velocity fluctuation of the PDC bit ranges from 8 to 20 rad/s, which makes the PDC bit less prone to stick-slip motion; however, the bit-bounce motion of the PDC bit occurs due to the fact that the minimum cutting resistance torque and dynamic weight on bit of the drill-bit are zero. Compared with the stick-slip motion in Fig. 4(b), the cutting resistance torque, dynamic weight on bit and axial displacement induced by the stick-slip and bit-bounce motions of the drill-string have more dramatic fluctuations. Therefore, increasing the speed of the rotary table is most detrimental to the security of the drill-string system in the case of the anomalous motion of the drill-bit. The dynamic characteristics of the drill-string system are shown in Fig. 4(d) while keeping the system parameters consistent with Fig. 4(c) and drilling the formation with a cone bit. In the stable phase, the rotation table has a velocity of 16 rad/s and the velocity fluctuation of the cone bit ranges from 12 to 20 rad/s. As a result, the stick-slip motion does not occur in the cone bit; The cutting resistance torque of the drill-bit and the dynamic weight on bit are much higher than zero, so the bit-bounce motion does not occur in the cone bit. Thus, when the drill-bit encounters soft formations in a shallow well, the cone bit can move normally. Compared with Fig. 4(b) and Fig. 4(c), the cutting resistance torque of the drill-bit, the dynamic weight on bit and the amplitude fluctuation range of the axial displacement are all small, so the normal drilling of the drill-bit is the key to ensure the safety of the drill-string.



(a) ω=43, b=1 (b) ω=74, b=1

(c) ω=118, b=1 (d) ω=118, b=3

Fig. 4 Dynamic characteristics of the drill-string system

5. Conclusion

(1) Reducing the rotational speed, weight on bit, and length of the bottom hole assembly facilitates normal drilling of the drill-bit when the PDC bit encounters soft formations in shallow wells, but makes normal drilling difficult when the PDC bit encounters soft formations in deeper wells. When the cone bit encounters soft formations in the shallow well, increasing the rotation speed, weight on bit and length of the bottom hole assembly favors normal drilling of the drill-bit; the high weight on bit is detrimental to normal drilling when the cone bit encounters hard formations in the shallow well.

(2) Coupled stick-slip and bit-bounce motions, as well as bit-bounce alone, induce more dramatic fluctuations in the cutting resistance torque, dynamic weight on bit, and axial displacement of the drill-bit compared to stick-slip motions. Thus coupled stick-slip and bit-bounce motions and bit-bounce alone are more harmful to the safety of the drill-string structure.

References

[1] Li ZF. Research and development of drill string mechanics. Oil Drilling. 2008.

[2] Tang L, Zhu X, Li J. Effects of the Synchronous Variation of the Static and the Kinetic Friction Coefficients on Stick–Slip Vibration of Drillstring. Iranian Journal of Science and Technology, Transactions of Mechanical Engineering. 2018;43(S1):275-83. doi: 10.1007/s40997-018-0156-y.

[3] Yigit AS, Christoforou AP. Stick-Slip and Bit-Bounce Interaction in Oil-Well Drillstrings. Journal of Energy Resources Technology. 2006;128(4):268-74. doi: 10.1115/1.2358141.

[4] Li ZF. Fundamental Equations and Its Applications for Dynamical Analysis of Rod and Pipe String in Oil and Gas Wells. ACTA PETROLEI SINICA. 1999(03):95-98+7-8.

[5] Jansen JD. Non-linear rotor dynamics as applied to oilwell drillstring vibrations. Journal ofSound and Vibration. 1991.

[6] Spanos PD, Chevallier AM, Politis NP. Nonlinear Stochastic Drill-String Vibrations. Journal of Vibration and Acoustics. 2002;124(4):512-8. doi: 10.1115/1.1502669.

[7] Batako ADL, Lalor MJ, Piiroinen PT. Numerical bifurcation analysis of a friction-driven vibro-impact system. Journal of Sound and Vibration. 2007;308(3-5):392-404. doi: 10.1016/j.jsv.2007.03.093.

[8] Zhu W. Modeling of Roller Cone Bit Lift off Dynamics in Rotary Drilling. SPE. 2011.

[9] Zhang H, Di Q, Wang W, Chen F, Chen W. Lateral vibration analysis of pre-bent pendulum bottom hole assembly used in air drilling. Journal of Vibration and Control. 2018;24(22):5213-24. doi: 10.1177/1077546317747778.

[10] Di QF, Hu FF. Dynamic behaviors of deviation control of the prebent pendulum BHA in gas drilling. Natural Gas Industry. 2019. doi: 10.3787/j.issn.1000-0976.2019.07.012.

[11] Fu M;Li JH. Nonlinear Self-Excited Vibration Mechanism for Under-Actuated Oil-Well Drilling System with Stick-Slip Vibration. Journal of Northwestern Polytechnical University. 2019;37(04): 802-808.

[12] Christoforou AP, Yigit AS. Fully coupled vibrations of actively controlled drillstrings. Journal of Sound and Vibration. 2003;267(5):1029-45. doi: 10.1016/s0022-460x(03)00359-6.

[13] Ritto TG, Aguiar RR, Hbaieb S. Validation of a drill string dynamical model and torsional stability. Meccanica. 2017;52(11-12):2959-67. doi: 10.1007/s11012-017-0628-y.

[14] Ritto TG, Ghandchi-Tehrani M. Active control of stick-slip torsional vibrations in drill-strings. Journal of Vibration and Control. 2018;25(1):194-202. doi: 10.1177/1077546318774240.

[15] Ritto TG, Soize C, Sampaio R. Non-linear dynamics of a drill-string with uncertain model of the bit–rock interaction. International Journal of Non-Linear Mechanics. 2009;44(8):865-76. doi: 10.1016/j.ijnonlinmec.2009.06.003.

[16] Kamel JM, Yigit AS. Modeling and analysis of stick-slip and bit bounce in oil well drillstrings equipped with drag bits. Journal of Sound and Vibration. 2014;333(25):6885-99. doi: 10.1016/j.jsv.2014.08.001.