ISSN: 1813-4890

Is Duopoly Less Competitive Than Monopoly?

Xingshuai Wang[1,*], Liang Shan[2]

[1] School of Economics, Shandong University of Technology, Zibo 255000, China

[2] Korea AI Blockchain Convergence, Seoul 06750, South Korea

[*]keen919@163.com

Abstract

We expands the dynamic duopoly of renewable resource in three different contracts of resource stock feedback under Cournot duopoly, monopoly equilibrium, linear feedback and nonlinear feedback strategies. Which shows under linear combination of output and linear combination of sales revenue contracts, the duopoly is more anti-competitive than monopoly as Fujiwara showed. Moreover, these results also valid that the price under monopoly is much more lower than other two feedback equilibrium. But under the contract of market share, the price of duopoly is much higher than nonlinear feedback strategies, which is contrast to the Fujiwara's conclusion.

Keywords

Cournot Duopoly; Feedback Strategies; Renewable Resources.

1. Introduction

This paper studies dynamic Cournot duopoly under three different contracts which is focus on output, sales revenue and market share. Tsutsui and Mino (1990) first absorbed sticky prices discuss nonlinear strategies in duopoly game which based on Fershtman and Kamien (1987) showed that duopoly has much more larger output than linear and nonlinear feedback. However, Fujiwara (2008) show that under the assumption of closed-loop Nash strategy (Rowat, 2007), among monopoly, static duopoly, linear and nonlinear feedback Nash solutions, linear feedback output turns out the smallest resource stock which is sharply contrasts with the Fershtman and kamien (1987) showed.

Lots of research (Lambertini, 2015 and 2016) on dynamic duopoly are based on open-loop Nash equilibrium. However, Benchekroun (2003) show that feedback under closed-loop Nash equilibrium strategies much better than open-loop.

To examine Fujiwara's issue, we propose an expanding dynamic duopoly game investigated by Benchekroun (2003), Fujwara (2008) under three different types of contracts. One is based on sales revenue, others are based on output and market shares. Compared with state-owned enterprises and large enterprises, the increase of commercial credit is more helpful to promote the technological innovation of private enterprises and small enterprises(Liu, 2021).

In this paper we extend three different types under closed-loop information, our main idea can be summarized as follow, based on the contracts of output and sales revenue. 1) The output under monopoly is much higher than both linear and nonlinear feedback in contracts. 2) Duopoly is much less competitive than monopoly, which is validated the Fujwara's conclusion. But under the contract of market shares, the price of duopoly is much higher than nonlinear feedback strategies, which is sharply contrast to the Fujiwara's conclusion.

The remainder of our paper can be organized as follows. In section 2 set up the model. In section 3 derives the linear and nonlinear feedback strategies under different types of contracts. In section 4 compares them among equilibriums and finally conclusion in section 5.

ISSN: 1813-4890

2. The Model

The duopoly model is an extension of Benchekroun (2003) and Fujiwara (2008). The market is supplied by two firms (firm 1 and firm 2) producing a homogeneous good. The inverse demand function is denoted by $p = a - X$, with $X = \sum_{i=1}^{2} x_i$, a>0 and p is the price. Both firms share the same technology and the marginal cost $c \in (0, a)$ constant over time $t \in [0, \infty)$. Both firms exploit the common renewable resource, which is described as:

$$\dot{S} = kS - X \tag{1}$$

Where S is the stock of resource, k>0 is its growth rate and $X = x_1 + x_2$, the ith firm maximizes the discounted sum of profits can be formulated as:

$$\int_0^{\infty} e^{-rt}\left(a - c - x_i - x_j\right)x_i dt, \quad r > 0, \tag{2}$$

Here r>0 is the discount rate. Common to both firm 1 and firm 2 constant over time.
Contract 1. Ritz (2008), linear combination of sales revenue is given by:

$$M_i = \left(a - c\varphi - x_i - x_j\right)x_i$$

Contract 2. Vickers (1985), linear combination of output is given by:

$$M_i = \left(a - c - x_i - x_j + \theta\right)x_i$$

Here we assume θ is constant and θ>0.
Contract 3. Linear combination on market share is given by:

$$M_i = \left(a - c - x_i - x_j\right)x_i + \frac{\mu^2}{2}$$

Here, μ is advertising effort.

3. Feedback Nash Equilibrium

The set up is following Fujiwara (2008), the firm i's Hamilton–Jacobi–Bellman can be written as:

$$rV(S) = \max_{x_i}\left\{\left[a - c - x_i - x_j(s)\right]x_i + V'(S)\left[kS - x_i - x_j(s)\right]\right\} \tag{3}$$

Where V (.) is firm i's value function.

3.1. Type-1 Contract

With the incentives for sales revenue, the firm i's Hamilton–Jacobi–Bellman can be rewritten as:

ISSN: 1813-4890

$$rV(S) = \max_{x_i}\{[a - x_i - x_j(s) - \varphi c]x_i + V'(S)[kS - x_i - x_j(s)]\} \quad (4)$$

The first order condition for firm i is:

$$V'(S) = a - \varphi c - 3x$$

Here we use the symmetry assumption of $x_i(S) = x_j(S) = x(S)$, substituting it into equation (6) and differentiating it, then we can get:

$$x'(S) = \frac{(k - r)[(3x(S) - (a - \varphi c)]}{8x - 3kS - (a - \varphi c)}$$

With the terminal condition $\lim_{t\to\infty} e^{-r}\ V(S) = 0$(Fujiwara, 2008), we can get:

$$x'(S) = 0 \leftrightarrow x(S) = \frac{a - \varphi c}{3}$$
$$\dot{S} = 0 \leftrightarrow x(S) = \frac{kS}{2}$$

3.1.1. Linear Feedback Strategy

Follow Fujiwara (2008) assumed, the strategy is $x(S) = \alpha S + \beta$, which is linear in S. And, extend it in our contact 1, we can get:

$$x'(S) = \alpha = \frac{(k - r)[3x(S) - (a - \varphi c)]}{8x(S) - 3kS - (a - \varphi c)}$$

Which we can rewrite as below:

$$\alpha = \frac{(k - r)[3(\alpha S + \beta) - (a - \varphi c)]}{8(S\alpha S + \beta) - 3kS - (a - \varphi c)}$$

From this equation, we can solve:

$$\alpha = \frac{6k - 3r}{8}$$
$$\beta = \frac{(-2k + 5r)(a - \varphi c)}{24k}$$

With the steady state condition, we can get the linear feedback Nash Equilibrium as:

$$S_1^L = \frac{(2k - 5r)(a - \varphi c)}{3k(2k - 3r)}$$

ISSN: 1813-4890

For the condition[14] k>5r/2, which means the renewable property is strong enough.

3.1.2. Nonlinear Feedback Strategy

Following Tsutsui and Mino (1990) and Fujiwara (2008), at the tangent point x'(S) =k/2, the equation x'(S) can be rewritten as:

$$\frac{k}{2} = \frac{(k-r)[\frac{3kS}{2} - (a-\varphi c)]}{kS-(a-\varphi c)}$$

Solve this equation by S, we have the nonlinear feedback as:

$$S_1^N = \frac{(k-2r)(a-\varphi c)}{k(2k-3r)}$$

So, it is easy to show that:

$$S_1^D = \frac{2(a-\varphi c)}{3k}$$

When the market is under monopolize, we can show that:

$$S_1^O = \frac{(a-\varphi c)}{2k}$$

3.2. Type-2 Contract

With the incentives for output, the firm i's Hamilton–Jacobi–Bellman can be written as:

$$rV(S) = \max_{x_i}\{[a-c-x_i-x_j(s)+\theta]x_i + V'(S)[kS-x_i-x_j(s)]\} \quad (5)$$

As it is relatively showed before, we can get the:

$$V'(S) = a-c-3x+\theta$$

And,

$$x'(S) = \frac{(k-r)[3x(s)-(a-c)-\theta]}{8x(S)-3kS-(a-c)-\theta}$$

As the same way before, we can get:

$$\alpha = \frac{6k-3r}{8}$$

$$\beta = \frac{(-2k+5r)[(a-c)+\theta]}{24k}$$

ISSN: 1813-4890

So, the linear and nonlinear feedback in contract-2 as below:

$$S_2^L = \frac{(2k-5r)[(a-c)+\theta]}{3k(2k-3r)}$$
$$S_2^N = \frac{(k-2r)[(a-c)+\theta]}{k(2k-3r)}$$

It is easy to show the duopoly and monopoly are:

$$S_2^D = \frac{2(a-c+\theta)}{3k}$$

And:

$$S_2^O = \frac{(a-c)}{2k}$$

3.3. Type-3 Contract

With the incentives for market share, the firm i's Hamilton–Jacobi–Bellman can be written as:

$$rV(S) = \max_{xi}\left\{\left[a-c-x_i-x_j(s)\right]x_i + \frac{\mu^2}{2} + V'(S)\left[\mu(kS-x_i-x_j(s))\right]\right\}$$

The same way, we can get:

$$V'(S) = \frac{r(a-c-3x)}{\mu}$$

And:

$$\mu = V'(S)(kS-2x)$$
$$S_3^N = \frac{6\mu x + a - c}{3\mu k}$$
$$S_3^D = \frac{2(a-c)}{3k}$$

4. Comparison of Different Equilibrium

As we calculated so far, what we want to do is comparison of different equilibrium among Cournot duopoly, monopoly equilibrium, linear feedback and nonlinear feedback strategies.

Under all assumptions and all we prepared so far, basic on the contracts under output and sales revenues, we can easily get that:

$$S_1^D > S_1^O > S_1^N > S_1^L,$$

And:

ISSN: 1813-4890

$$S_2^D > S_2^O > S_2^N > S_2^L$$

Lemma 1.
Under contracts of output and sales revenues, we can show $S^D > S^O > S^N > S^L$.
Here, S^D is denoted by stock of resource under duopoly, S^O is denoted by the stock of resource under monopoly.
Lemma 2.
Under contracts of market share, we can show $S^N > S^D$.
Proof. It is easy to show that:

$$S^D = \frac{2(a-c)}{3k}$$

As:

$$x = \frac{2(a-c)}{3}$$

We can easy show that:

$$S_3^N > S_3^D$$

5. Conclusion

Here in this paper we revisited dynamic renewable resource duopoly proposed by Fujiwara (2008) and extension it into different three types of contracts. One is based on sales revenue, the others are based on output and market share. Compared among the equilibriums, we can valid that under contract of market share, the duopoly is not less competitive which is sharply contrast to Fujiwara's conclusion. This most main contribution of this paper is to show that the conclusions of Fujiwara (2008) is not applicable for all of the contracts.

Acknowledgments

This work was supported in part by the Soft Science Foundation of Shandong Province under Grant 2020RKB01348.

References

[1] Benchekroun, H. Unilateral production restrictions in a dynamic duopoly, Journal of Economic Theory, vol. 111(2003), 214–239.

[2] Benchekroun, H. Comparative dynamics in a productive asset oligopoly, Journal of Economic Theory, vol. 138(2008), 237–261.

[3] Dockner, E.J., Sorger, G. Existence and properties of equilibria for a dynamic game on productive assets, Journal of Economic Theory, vol. 71(1996), 209–227.

[4] Fershtman, C., Kamien, M. Dynamic duopolistic competition with sticky prices, Econometrica, vol. 55(1987), 1151–1164.

[5] Itaya, J., Shimomura, K. A dynamic conjectural variations model in the private provision of public goods: a differential game approach, Journal of Public Economics, vol. 81(2001), 153–172.

[6] John Vickers. Delegation and the Theory of the Firm, The Economic Journal, vol. 95(1985), 138-147.

ISSN: 1813-4890

[7] Lambertini L. Managerial Delegation in a Dynamic Renewable Resource Oligopoly, Dynamic Modeling and Econometrics in Economics and Finance, vol. 22(2016), 93-107.

[8] Robert A. Ritz. Strategic incentives for market share, International Journal of Industrial Organization, vol. 26(2008), 586-597.

[9] Rowat, C. Non-linear strategies in a linear quadratic differential game, Journal of Economic Dynamics and Control, vol. 31(2007), 3179–3202.

[10] Rubio, S.J., Casino, B. A note on cooperative versus non-cooperative strategies in international pollution control, Resource and Energy Economics, vol. 24(2002), 251–261.

[11] Shimomura, K. The feedback equilibria of a differential game of capitalism, Journal of Economic Dynamics and Control, vol. 15(1991), 317–338.

[12] Tsutsui, S., Mino, K. Nonlinear strategies in dynamic duopolistic competition with sticky prices, Journal of Economic Theory, vol. 52(1990), 136–161.

[13] Liu Tinghua: Study on the Influence of Commercial Credit on Technological Innovation of Enterprises(China Social Sciences Press, China 2021).

[14] Benchekroun (2003) and Fujiwara (2008) also require a similar condition.