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Better Planned Power Curve of a Cyclist

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Abstract

In the individual time trial of bicycle road races, the winner is the rider who rides a fixed course alone in the least amount of time. This paper creates a power curve model that helps the rider to minimize the time required to cover a given distance. In order to focus on the determination of rider's total energy as well as consider defining recovery power and recovery time, omni power duration model is adopted. To determine the relationship between the position of the rider on the track and the power applied by the rider, we established an optimal strategy model to programme the power distribution of riders dynamically. Additionally, we considered the potential impact of weather conditions, including wind directions and wind strengths, to determine our model's sensitivity to small differences in the weather. Wind direction and wind force affect the result of the game by forming wind resistance. On this basis, the speed of different riders under different tracks can be corrected. The change proportion of results caused by different wind conditions is calculated as the sensitivity of the competitions results to small differences in weather and environment. What's more, for the purpose of determining how sensitive the results are to rider deviations from the target power distribution, the climb time is shortened to observe the changes in the power curve. The sensitivity of the model is measured by race time bias. A conclusion is drawed that the sensitivity of the model is good since that there is little difference between the finish times.

Keywords

Power Curve Model; Omni Power Duration Model; Optimal Strategy Model.

1. Introduction

With the popularity of bicycle road races, models need to be built to measure rider's chance of success which depends on the type of event. So, reports point out the best strategies for different types of racers to minimize the time required to cover a given distance. Firstly, we establish a power curve model based on the four values which reflects the capability of the rider. Then we apply our model to three tracks, two of which are actual race tracks, and the last is the track we design ourselves. By utilizing optimal strategy mode, we determine the optimal power distribution of different tracks. Furthermore, we consider the potential impact of weather conditions, including wind director and wind strengths, to determine our model's sensitivity to small differences in the weather. Last but not least, we evaluate model's sensitivity to riders' deviations from the power targets.

2. Preperation of the Models

2.1. Assumptions and Justifications

- There is no difference between the racers' strength level.
- The wind direction and the wind are stable, which means that there is no random case.
- There is an upper limit to the rider's total energy.

- The recovery power is the critical power, and the recovery time is 3 min.

2.2. Notations

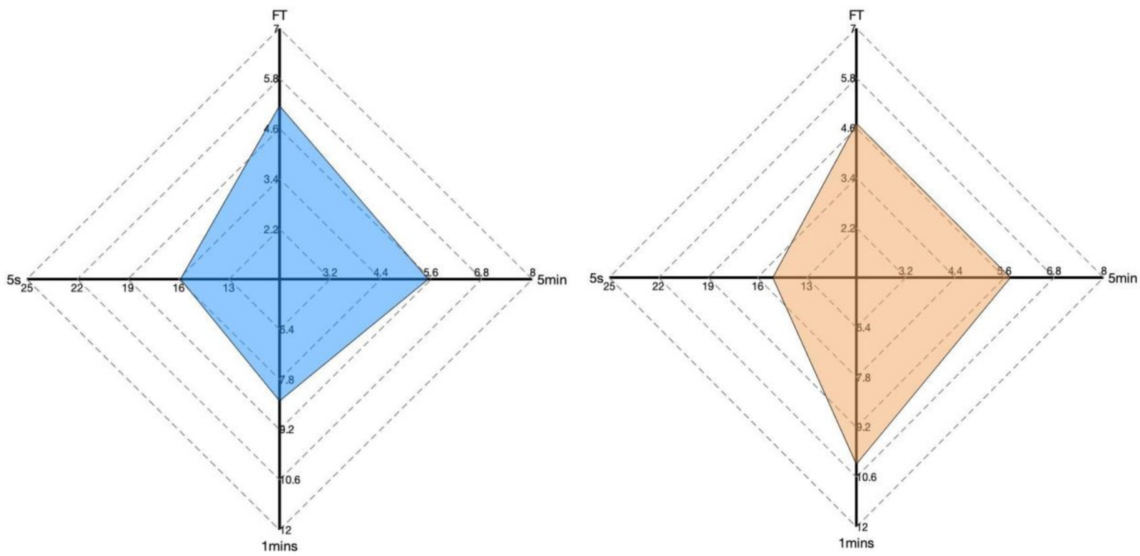
Table 1. Notations

Symbol	Definition
P	Power Output
W'	Work Above Critical Power
P_{max}	1s Peak Power
CP	Critical Power
t	Time in Seconds
π	An Optimal Strategy
CP_{TTF}	Time to Task Failure at Critical Power
V	A Fixed Constant for the Decline in Power Output Over Time
St	Current Slope Position of the Rider and the Current Environment
A_t	The Force Applied by the Rider at t
$V(S_t)$	The State Value of the Rider in the State S_t of the Journey

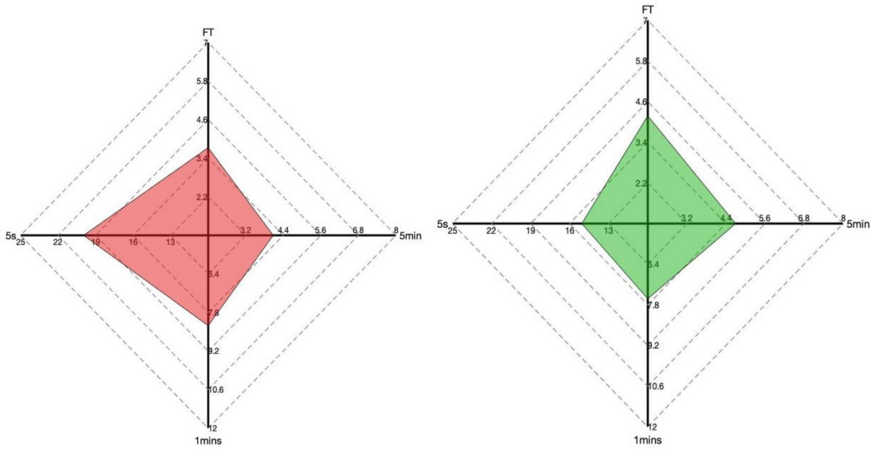
3. Power Curve Model

3.1. Rider Capability Definition

There are four very important values tell us about the rider’s capability, including sprint abilities(5s), anaerobic capcibility described by 1minute maximum power, 5 minutes to tell us about VO2 max capcibility and 20 min to describe our FTP [1]. Figure 1 reflects talent in cycing of four types of riders.



(a) (b)



(c) (d)

Figure 1. The capability of four types of riders

3.2. The Power Profiles of Two Types of Riders

We determine a preliminary model based on the four values mentioned aboved. And the power curve [2] which is represented by a line graph is a visual description of the maximum power generated by the rider over a period of time. The X axis represents the time and the Y axis represents the circulating power in watts. Since large power lasts only a short time [3], the power curve is usually tilted downward to the right.

However, the preliminary model assumes that a given power output is infinitely sustainable by default [4], while this is clearly not the case for real-world performances where exercise at the CP [5] is limited to 20–40 min. So, we adopt an Omni power duration model [6] to estimate the change in power over time. The formula is as follows:

$$P(t)=\frac{W'}{t}\times(1-e^{-t\times\frac{P_{max}-CP}{W'}})+CP;t\le CP_{TTF} \quad (1)$$

$$P(t)=\frac{W'}{t}\times(1-e^{-t\times\frac{P_{max}-CP}{W'}})+CP-A\times Ln(\frac{t}{CP_{TTF}});t>CP_{TTF} \quad (2)$$

In the formula, W' is defined as the work above critical power, and P_{max} is 1s peak power as well as CP is critical power. Other parameters like t are time in seconds, and CP_{TTF} is the time to task failure at critical power. A represents a fixed constant for the decline in power output over time. Therefore, the final power profiles are shown in the figures below:

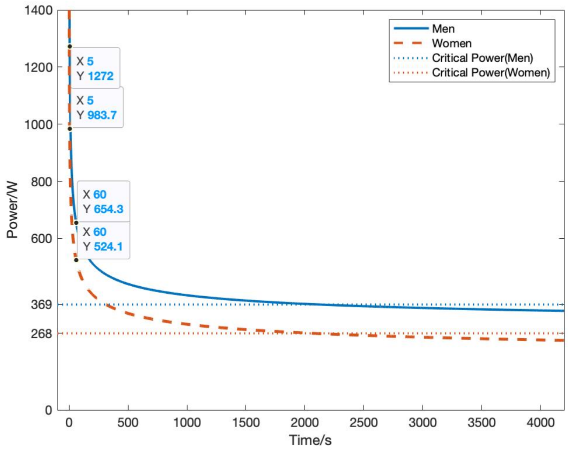


Figure 2. The power curve of the time trial specialist

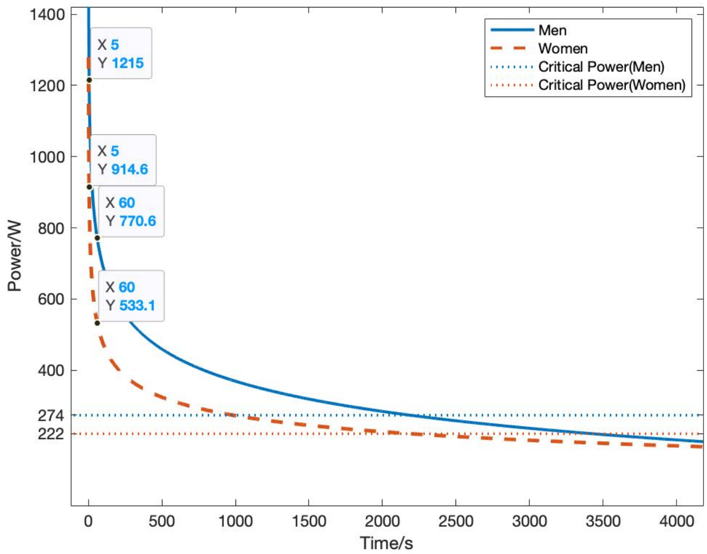


Figure 3. The power curve of the climber

3.3. The Power Profiles of Other Types of Riders

We also consider profiles of riders of different genders. As the figures below show, the output power of women is generally lower than that of men, which indicates that the total energy of male riders is higher than that of female riders.

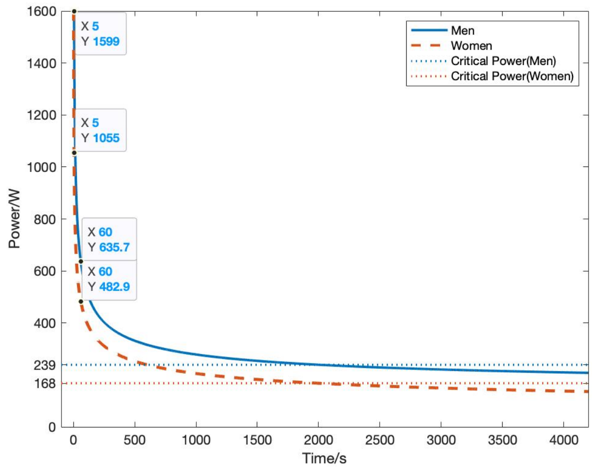


Figure 4. The power curve of the sprinter

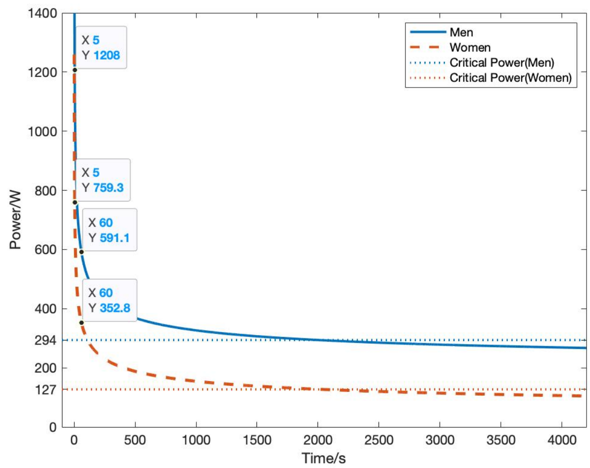


Figure 5. The power curve of the allrounder

4. Application of Model

4.1. Track Simplification

We simplify the slope of the first two actual race tracks in order to solve the problem. The following is a visual display of the simplified profile view.

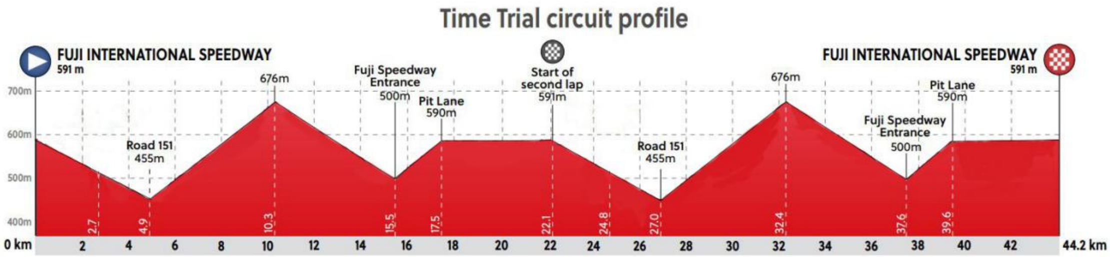


Figure 6. The simplified profile view of 2021 Olympic time trial course

We calculate the maximum slope of the time trial course in Flandersk, and the result is 0.9%. Consequently, we simplify the whole track into a flat track without slope.

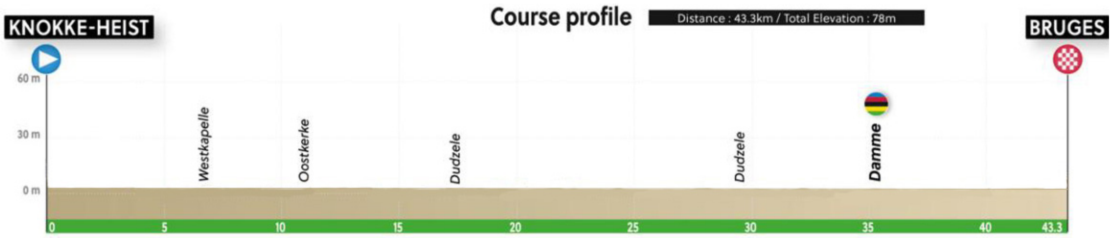


Figure 7. The simplified profile view of 2021 UCI World Championship

4.2. Optimal Strategy Model

In order to determine the relationship between the position of the rider on the track and the power applied by the rider, we establish an optimal strategy model to programme the power distribution of riders dynamically.

We set the current slope position of the rider and the current environment as an array or tensor, which is recorded as the state St. The force applied by the rider at t can be represented by action A_t, so our problem is equivalent to finding an optimal strategy π to make us reach the destination in the shortest time, that is, we should give certain punishment to each process, If the destination is not reached, a return R = - 1 will be fed back, and the value of the rider's state can be modeled and expressed as:

$$\max_{\pi} E[\sum_{t=0}^{H} \gamma^{t} R(S_t, A_t, S_t + 1) | \pi] \tag{3}$$

We use the time difference method TD (a combination of Monte Carlo and dynamic programming) in order to solve the above formula, and use Bellman equation to solve the optimal strategy.

We define the state value of the rider in the state S_t of the journey as $V(S_t)$, and the action state value $Q(S_t, A_t)$ represented by action A_t. In order to solve the optimal strategy π, we introduce the equation:

$$V_{\pi}(s) = E_{\pi}[R_{t+1} + \gamma V(s') | S_t = s] \tag{4}$$

$$Q_{\pi}(s,a) = E_{\pi}[R_{t+1} + \gamma Q_{\pi}(S_{t+1}, A_{t+1}) | A_t = a, S_t = s] \tag{5}$$

The optimal strategy is actually to find the optimal value action function Q * (s, a) = MAX $_{\pi}$Q * (s, a):

$$Q^*(s,a)=\sum_{s'}P(s'|s,a)(R(s,a,s')+\gamma \max_{a'} Q^*(s',a')) \tag{6}$$

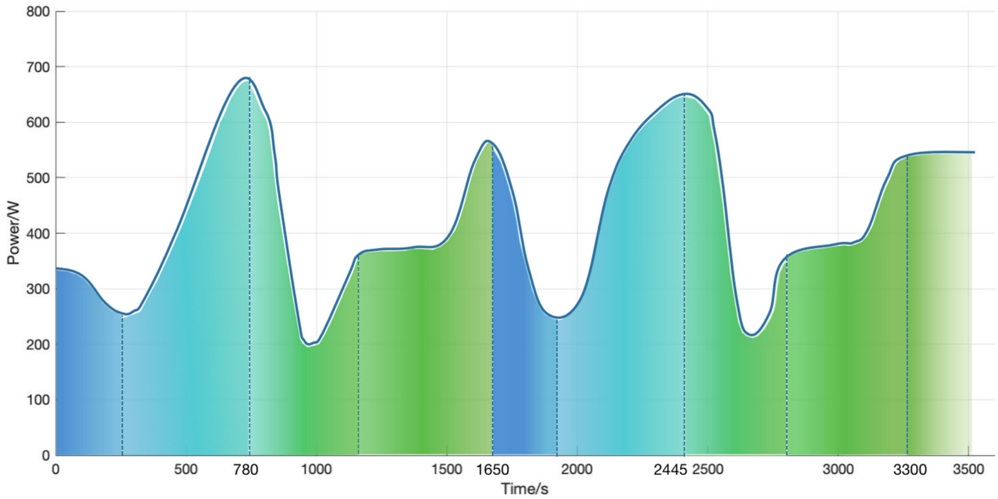


Figure 8. Optimal power distribution of 2021 Olympic time trial course

According to the above figure, it's clear to us that the downhill after each uphill will enter the recovery stage. When entering the flat slope section, the rider shall reserve enough energy for sprint.

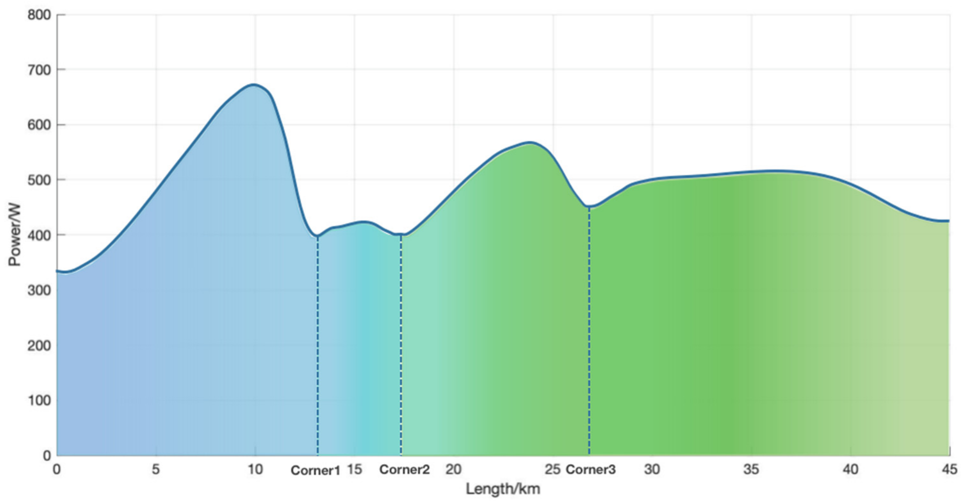


Figure 9. Optimal power distribution of 2021 UCI World Championship time trial course

From the topographic profile above, we know that the slope has almost no effect on the rider's power output. As a result, we only consider the influence of sharp turns on the power output distribution.

4.3. Self-designed Track

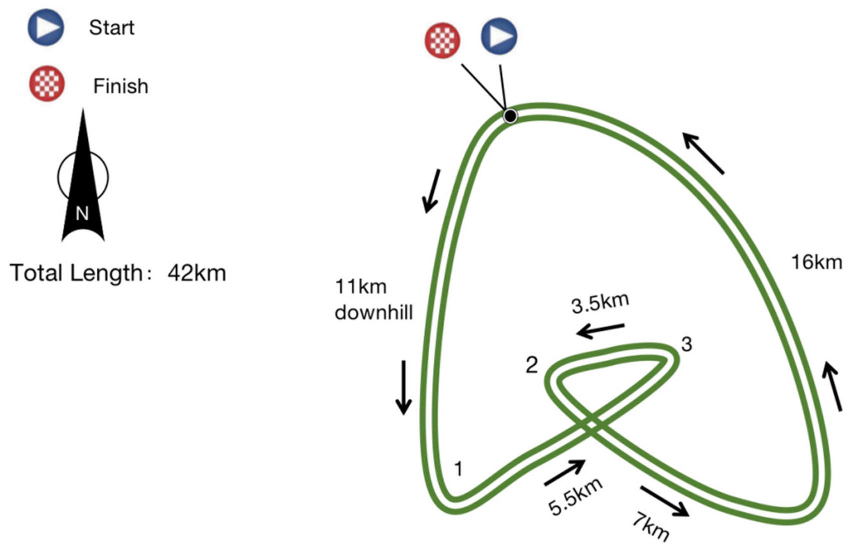


Figure 10. Track we design ourselves

The track we design includes four sharp turns and a downhill section. From the starting point, the rider will go through a downhill of 11km and then enter the flat road section as well as the first sharp turn. The second sharp turn will come at 5.5km from the beginning of the flat road section, and the third sharp turn is 3.5 km from the second sharp turn. Additionally, the last one is 16 km from the starting point which is also the terminal point.

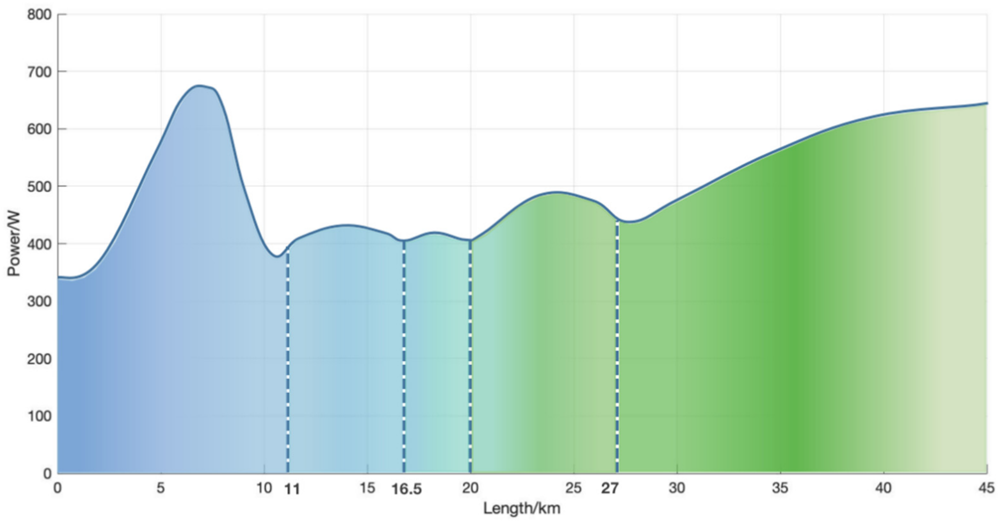


Figure 11. Optimal power distribution of the track we design ourselves

5. Sensitivity of the Model to Small Differences in the Weather

5.1. Representation of Wind Resistance

Wind direction and wind force affect the result of the game by forming wind resistance. We assume that the wind direction and wind force are stable. On this basis, the speed of different riders under different tracks can be corrected. The change proportion of results caused by different wind conditions is calculated as the sensitivity of the competitions results to small differences in weather and environment.

According to the principle that the faster the speed is, the wind resistance increases exponentially. When the speed driven by the power allocated by the rider at different positions is uneven, the impact is greater. On the contrary, considering the extreme situation, no matter what road, when the whole process is at a constant speed, it will be less affected. The calculation formula of air resistance is as follows:

$$F=(1/2)C\rho\nu^{2} \tag{7}$$

In the formula, C is the air resistance coefficient; ρ is the air density which is 1.293 kg/m^3; S is windward area of object; ν is the relative velocity of the object and air.

$$P=(F-f)v \tag{8}$$

In the formula, P is actual output power; v is the relative velocity of the object and air; f is wind force.

5.2. Wind Grade Representation

Table 2. Wind scale

Wind Scale	Designation	Wind Velocity(m/s)	(km/h)
0	No Breeze	0.0-0.2	0-0.7
1	Light Breeze	1.6-3.3	6-11
2	Soft Breeze	5.5-7.9	20-28

We define the level of wind that may appear in the game as five levels. Different wind speeds correspond to different wind scales. By utilizing the wind resistance model above, we can easily find the resistance thus getting race times. We average the five race times and then express the sensitivity with variance. The final variance is 1.655 min which proves that the sensitivity of the model is good.

6. Model's Sensitivity to Riders' Deviations from the Power Targets

In order to determine how sensitive, the results are to rider deviations from the target power distribution, we shorten the climb time to see what happens to the power curve. The sensitivity of the model is measured by race time bias. The following two graphs compare the power curves before and after the deviation from the target.

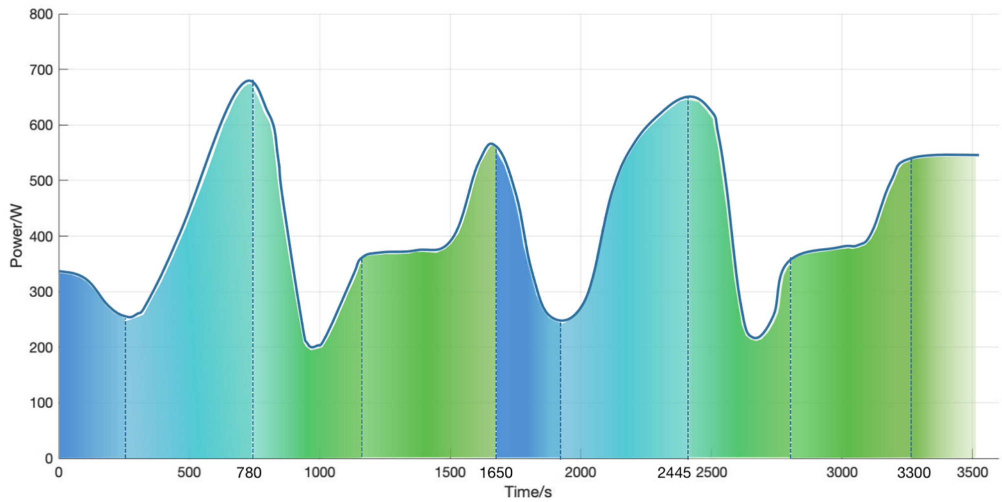


Figure 12. The power curves before the deviation from the target

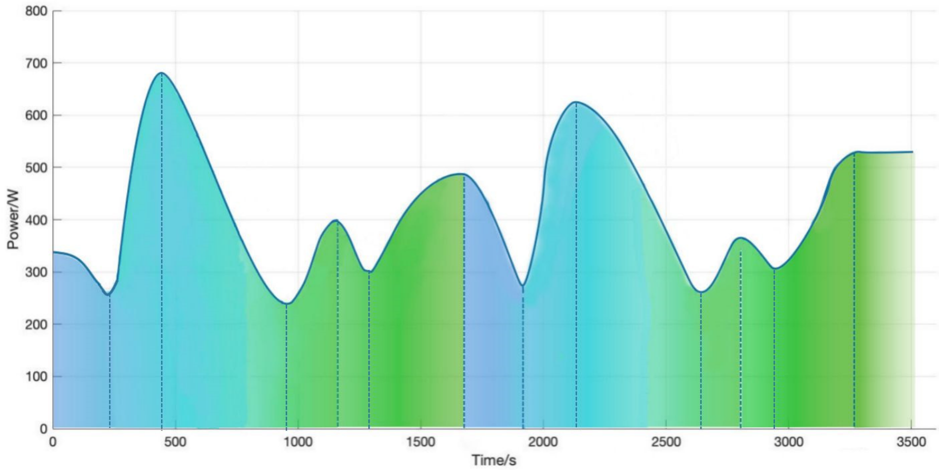


Figure 13. The power curves after the deviation from the target

We can draw a conclusion from the picture that the sensitivity of the model is good since that there was little difference between the finish times.

7. Extend the Model to a Team Time Trial

From individual time trials to team time trials, the number of riders has increased from one to six, and the allocation of different types of riders needs to be considered. Since the final result of the team race is the result of the fourth arriving rider of the team, and in the previous model, we analyzed the competition decision of the individual time trial contestants, the last two riders are preliminarily considered in the decision-making for the early wind breaking sprint to establish the preliminary advantage, After the first two charge, select professional time trial experts and generalists to participate in the later competition. Therefore, we can further configure the formation of the team with the help of the model. When the bicycle is running, because the air in front is compressed, the friction between the surfaces on both sides and the air and the space behind the tail become part of the weak resistance space. Therefore, in team competition, we can arrange the position (front or rear) of the team according to the type of team members, arrange the sprinters at the back of the team, reduce the wind resistance, take the front players to the best sprint position, and start sprinting to shorten the time. In addition, since the team performance depends on the fourth player in the team, we need to combine the types of players and the corresponding power curve to determine the formation transformation in the final sprint stage.

8. Conclusion

The power curve model created does help the rider to minimize the time required to cover a given distance. Taking the potential impact of weather conditions into consideration, our model's sensitivity to small differences in the weather is good. What's more, with regard to how sensitive the results are to rider deviations from the target power distribution, a conclusion is drawed that the sensitivity of the model is good since that there is little difference between the finish times.

Acknowledgments

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